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Le guide est un complément au programme de mathématiques du premier cycle du secondaire. Son contenu n'est pas obligatoire. L'intention générale du document est de favoriser chez l'enseignant une meilleure compréhension du programme et de suggérer des méthodes d'enseignement et des activités d'apprentissage adaptées au programme. Les chapitres abordent les objectifs généraux, terminaux et intermédiaires du programme, la question de l'éducation et le matériel didactique requis. Une bibliographie sélective complète le tout.

Type de document: d

Curriculum Guide

SECONDARY SCHOOL

MATHEMATICS

FIRST CYCLE

Québec

Curriculum Guide

SECONDARY SCHOOL

MATHEMATICS

FIRST CYCLE

Direction générale des programmes
Direction de la formation générale

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1. INTRODUCTION

This Curriculum Guide is a supplement to the Mathematics Program for the first cycle of secondary school (Document No. 16-3301A). Its content is not compulsory and the document itself is being suggested only as an optional teaching aid.

> "Its general intention is to promote a better understanding of the program, to suggest certain teaching approaches and learning activities which are considered suitable for an effective implementation of the program. It also aims to encourage teachers to be creative in their search for pertinent and effective teaching techniques."[1]

This document contains a number of notes on each of the general, terminal and intermediate objectives. These notes and commentaries are useful guidelines for planning courses. Their first objective is to define scope, to suggest approaches, and to propose strategies. They are consistent with the major principles of methodology and adhere to the guiding principles which governed the development of the program.

In several places the Curriculum Guide suggests a number of enrichment items likely to enable motivated pupils to explore more thoroughly certain areas covered by the program. In no case, however, should these enrichment items be drawn from the content of the Mathematics Program for the second cycle of secondary school.

---

1. D.G.D.P., Cadre relatif à l'élaboration des programmes et des guides pédagogiques, 1980. This citation is a free translation.

It should also be stressed that the Curriculum Guide is not a "lesson plan," and that the information it contains should not be transmitted directly to the pupil. The notes and commentaries are intended for the teacher, not for the pupil. It should be expected, therefore, that some materials are treated in greater detail in this guide than those found in materials intended for presentation to pupils.

It is essential to ensure that all pupils have a good grasp of this basic program, and about 75% of the time available should be devoted to this purpose. This leaves enough room for remedial work for the weaker pupils, and for enrichment activities based on pertinent materials for the more advanced pupils. It is the responsibility of the schools to define the content of the remedial and enrichment activities.

The use of problem-solving techniques is stressed throughout this guide to highlight the practical aspect of mathematics and to unify the different themes developed in the program (numbers, measures, location, geometry, statistics, and probability). Also, in the context of a basic Mathematics program, the importance of cultivating operational skills should not be underestimated. A good understanding of the basic concepts does not make the acquisition of reliable and effective algorithms superfluous.

Far from neglecting the contribution of the new technologies, it is important to examine carefully the impact of the pocket calculator on society, particularly in the schools. Many pupils own one and want to use it. Every teacher should adjust his methodology so as to facilitate the rational use of this tool in the classroom. Indeed, one of the guiding principles of the first cycle program encourages "the use of the hand calculator in situations other than those intended to develop basic operational skills."[1]

1. M.E.Q.P. Secondary School Curriculum - Mathematics First Cycle (Document 16-3301A).

The statement of this principle itself shows the spirit in which the calculator should be used in the classroom. It is to be a tool for helping the pupil to have a better understanding of the algorithms employed. In no case should the calculator hamper the acquisition of basic operational skills. However, the contexts in which it may be used are many:

- finding patterns of numbers;
- evaluating a sequence of operations on large or small numbers;
- solving equations by an iteration method;
- validating approximations;
- checking answers;
- etc.

The chapter on educational evaluation suggests, as an example, a form which can facilitate the continuous evaluation of the pupil's learning. Such an evaluation enables the teacher to adapt his remedial teaching activities to the particular needs of each pupil (form "A"). Another form, form "B", could eventually serve as an evaluation tool which could help school authorities to make decisions respecting the promotion of each pupil.

For additional information the teacher should consult Chapters 4 and 5 ("Instructional Materials" and "Bibliography"). The information given in these two chapters is far from being exhaustive and may be supplemented by an inventory of local resources.

In order to standardize the various graphs, notations, and symbols, and to make their interpretation unequivocal, the teacher should consult the Information Document - Graphs, Notation, and Symbols Used in Secondary Mathematics (Document 16-3306A). This list will be used in preparing the tests or examinations of the ministère de l'Education.

2. TEACHING NOTES

GENERAL OBJECTIVE

1

TO ENCOURAGE pupils to apply their knowledge of natural numbers or integers.

THEME: Natural numbers and integers

TIME: 14% in Secondary I
8 % in Secondary II

In elementary school, a minimum of 40% of the time is allocated to the study of natural numbers. Arithmetic

> "is to a certain extent the foundation on which the greater part of the elementary Mathematics program is built."
>
> "Teachers should seek to develop activites which involve exploration, abstraction, and the gradual application of the major concepts of natural numbers in various simple situations drawn from real life or from the pupil's experience. Similarly, teachers should try to encourage the acquisition of operational and related mental skills in order to facilitate the learning of other systems of numbers."
>
> "Teachers should also strive to help the pupil to develop a certain competence in estimation."[1]

The general objective, "TO ENCOURAGE pupils to apply their knowledge of natural numbers or integers," should make it possible to study in greater detail certain applications of natural numbers, as it allows the pupil to have a better grasp of the system of numeration and of operations on natural numbers.

---

1. M.E.Q., Programme d'étude primaire, mathématique, Chapter 3. This passage is a free translation.

Using situations drawn from real life should help ensure continuity between the programs in elementary and secondary school. It should also promote the development of heuristics, such as problem-solving strategies.

It should be pointed out that it is not the intention of this program to make a detailed study of integers. However, the scope of the study undertaken should be sufficient to enable the pupil to better interpret reality and to develop problem-solving heuristics, giving due consideration to the advantages of the rational use of the calculator.

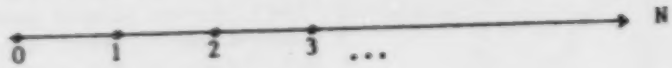
TERMINAL OBJECTIVE 1.1 TO SOLVE problems dealing with numeration.

This objective should enable the pupil to explore in greater depth the concepts learned in elementary school. The activities dealing with numeration should be integrated into situations which are likely to highlight the utilitarian role of mathematics.

In covering this objective, it is possible and even desirable to devise a comprehensive activity involving all the intermediate objectives. Thus, an activity such as regrouping toothpicks can lead to a study of the following objectives:

- to read a number,
- to write a number,
- to compare several numbers,
- to determine a place value and the number of tens, hundreds, etc.,
- to write a number in expanded form,
- to round off a number.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
1.1.1 TO READ a natural number greater than one million.	I	- In the second cycle of elementary school the pupil should have learned "to read and to write all numbers less than 1 000 000." (Objective 2.5). - Review grammar rules respecting the writing of numbers. - Application: writing out numbers on cheques, receipts, bills, contracts, etc. In general, a natural number is expressed in base ten using a set of digits. These are: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. There are, however, other ways of writing natural numbers: - The Babylonians used cuneiform writing to represent numbers; - the Egyptians used hieroglyphics or hieratic writing to represent numbers; - the Greeks invented the Attic system to represent numbers; - the Romans used the Roman system to represent numbers; - the Chinese developed their own system of numeration; - etc. Since the Roman system is used for writing centuries, years, or hours, and also for numbering the first pages of certain books, etc., it is suggested that the teacher discuss this system and its principal numerals (I, V, X, L, C, D, and M).
1.1.2 TO WRITE a natural number greater than one million.	I	

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
1.1.3 TO COMPARE two natural numbers.	I	- Present the symbols $<$, $=$, and $>$. - Place numbers on a natural number line:  - Compare: - masses; - prices; - lengths. - In the first cycle of elementary school the pupil should be able to: "Arrange a set of numbers in ascending or in descending order; "Find the number that comes immediately before or immediately after a given number, or the number between two others; "Determine the position of an element in a set of numbers in which the elements are arranged in a given order."
1.1.4 TO ESTABLISH the place value relationships in a natural number.	I	It is advisable, at this point, to explore further the concept of order in natural numbers and to relate that concept to the symbols currently in use ($<$, $=$, and $>$). - Present $N = \{0, 1, 2, 3, \ldots\}$ - Make a clear distinction between "digit" and "number." - Consider only cases in which numbers are expressed in base ten. - In the first cycle of elementary school two intermediate objectives make use of the concept of base: "To group and to regroup objects in different bases."

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		"To associate a given number with a set of elements grouped in a given base." By contrast, in the second cycle of elementary school the concept of "base" is no longer mentioned. The pupil, however, should be able "to compose or to break down a number expressed in base ten" (Terminal Objective) if he attains the following intermediate objectives: "To round a number to a given position of place value. "To determine the value of a digit or a group of digits in a number according to its place. "To identify the value of each place in a number." - The place value of a digit in a number depends on the system of numeration used. Example: - decimal system of numeration: the place value of the digit "5" in the number 3547 is: 5×10^2, or 5 x 100, or 500. - octal system of numeration: the place value of the digit "5" in the number 3547 is: 5×8^2, or 5 x 64, or 320. However, in base 10 this expression is equivalent to: 5×10^2, or 5 x 100, or 500. - Concrete applications can be found for this objective. Example: If you have 13 419 dollars, how many - 10$, - 100$, - 1000$, do you have?

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		Example: If you buy 13 419 candies, how many - boxes of 10 candies, - boxes of 100 candies, - boxes of 1000 candies, will you have?
		Example: In the number 13 419, there are - 1341 tens, - 134 hundreds, - 13 thousands, etc.
1.1.5 TO WRITE a number in expanded form.	I	- This objective should enable the pupil to express the number 1249 thus: $1249 = (1 \times 1000) + (2 \times 100) + (4 \times 10) + (9 \times 1)$ or $= (1 \times 10^3) + (2 \times 10^2) + (4 \times 10^1) + (9 \times 10^0)$. - The concept of power has been presented to the pupil in the second cycle of elementary school: "To express a product of identical factors in exponential form, and vice-versa."
1.1.6 TO ROUND a number to a given degree of accuracy.	I	- Rounding a number is often a preliminary step to making an estimation. In the number 5764 the highest value, that is, the place of the thousands, should be determined first. As 5 is followed by a 7 and the nearest thousand is 6, therefore 5764 gives 6000 as a value rounded off to the nearest thousand.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- By similar reasoning it is possible to state that 5764 gives 5800 as a value rounded off to the nearest hundred, 5760 as a value rounded off to the nearest ten. - It is not necessary to explain at length the two rounding off techniques by excess or by default. - Rounding skills become increasingly essential particularly if the pupil wishes to check quickly the correctness of certain calculations done on the calculator. Example: If the pupil uses the calculator to compute 5472 x 48, he will know that the value displayed is reasonable if it is close to: 5500 x 50 = 275 000. Indeed, if the values 5472 and 48 are entered correctly into the calculator, the machine should display the number 262 656. As this number is relatively close to 275 000, it can be assumed that the result displayed is accurate.

TERMINAL OBJECTIVE 1.2 TO PERFORM basic operations on natural numbers using various algorithms.

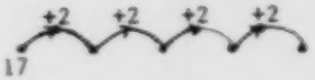
The primary purpose of this objective is to strengthen the knowledge and skills developed in elementary school. Mastery of the addition and multiplication tables can be ensured by using several commercial games, through activities in which the calculator is used as a verification tool, through activities such as modular arithmetic linking mathematics and art, and through the application of the properties of operations, in brief, repetitive exercises.

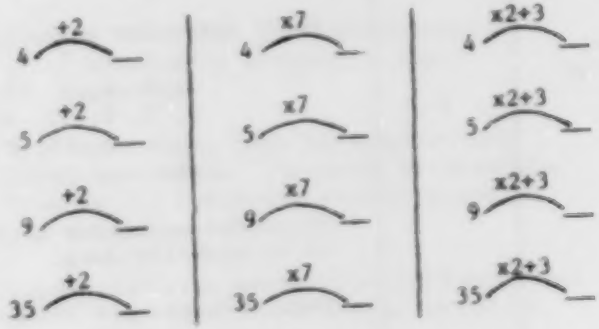
This objective also promotes the use of strategies. The application of "problem-solving" techniques is strongly recommended. Consequently, it is important to systematize the process involved. Its major stages can be outlined in the following way:

1. Reading the problem carefully.
2. Restating the problem in your own words.
3. Selecting the important data of the problem.
4. "Mathematizing" the situation (constructing a mathematical model).
5. Evaluating the result.
6. Solving the problem.
7. Interpreting the result.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
1.2.1 TO DETERMINE mentally the result of a chain of operations involving natural numbers.	I	- The development of mental calculation skills is the focus of one terminal objective in the second cycle of elementary school: "To perform mental and written operations or series of operations on natural numbers." - Use only simple exercises. Example 1: 13 + 67. Example 2: 250 + 350. Example 3: 73 - 22. - Pupils should be given a chain of operations arranged in any given order. Examples: 11 + 25 + 19 + 5 ; 3 x 4 + 5 - 2 - 5 11 + 19 + 25 + 5 ; 12 - 2 + 5 - 5 30 + 30 = 60 ; 10 + 0 = 10 - The observation and regrouping or utilization of the properties of natural numbers are essential for the acquisition of a certain speed in mental calculation.
1.2.2 TO DETERMINE the sum, the difference, the product, or the quotient of natural numbers.	I	- In the second cycle of elementary school the pupil should be able: "To add two or more numbers whose sum is less than 100 000." "To find the difference between two numbers less than 100 000." "To find the product or the quotient of a number less than 1000 multiplied or divided by a number less than 100." - In Secondary I, the result of these operations should be less than 100 000 000.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- It is advisable to vary the calculation techniques: - usual algorithms of operations; - utilization of: - the commutative property, - the associative property, - the distributive property. - The calculator should be used as a means of checking calculations. - If the need arises, the teacher should point out to the pupil that the +, −, x, and ÷ operations respect a certain order of priority. Examples: 2 + 3 x 4 equals 14, not 20. 6 − 2 ÷ 2 equals 5, not 2.
1.2.3 TO ESTIMATE the result of an operation by the rounding off technique.	I	- It is a question of systematizing the pupil's ability to estimate the degree of accuracy of an answer by rounding the given values and then performing the operations required. Example: 253 x 11 + 1308 gives, by rounding, 250 x 10 + 1300 = 2500 + 1300 + 3800 Estimate: 3800. Example: $\frac{29 \times 102}{20}$ gives, by rounding $\frac{30 \times \cancel{100}^{5}}{\cancel{20}_{1}} = 150$. Estimate: 150.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
1.2.4 TO PERFORM computations on natural numbers involving the order of operations.	I	- Depending on the case, the following should be discussed: — order of operations, — the role played by parentheses in modifying this order, — the distinction between term and factor. Examples: 1.  17 + 2 + 2 + 2 + 2 = . . . 2. (17 + 2) x 3 - 30 + 18 = . . . 3. When the proper order of operations is followed, 2 + 3 x 4 equals 14, whereas (2 + 3) x 4 equals 20. - The use of the calculator should provide an opportunity to stress the necessity of entering an expression accurately and of following strict writing rules in composing a mathematical expression. - It is strongly recommended that situations drawn from the pupil's experience be used. Here is an example: You wish to share a certain number of marbles equally among your six friends. If you have 3 bags of 20 marbles, 4 bags of 18 marbles, and 5 bags of 12 marbles, how many marbles will each of your friends have? $(3 \times 20 + 4 \times 18 + 5 \times 12) \div 6 =$ $\frac{60 + 72 + 60}{6} =$ $\frac{192}{6} = 32.$ - ENRICHMENT: - To present more complex expressions containing one or two levels of parentheses.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
1.2.5 TO ASSOCIATE a power of a natural number in its exponential form, and vice-versa.	I	- Explore in depth the concept of power. - This concept has been presented in the second cycle of elementary school: "To express a product of identical factors in exponential form, and vice-versa." Examples: $5^2 = 25$ as $5^2 = 5 \times 5$ $3^3 = 27$ as $3^3 = 3 \times 3 \times 3$ $3^5 = 243$ as $3^5 = 3 \times 3 \times 3 \times 3 \times 3$ - It is possible to find the prime factors of a number and express the result using exponents. Examples: $24 = 2^3 \times 3^1$ $42 = 2^1 \times 3^1 \times 7^1$ $45 = 3^2 \times 5$.
1.2.6 TO APPLY a function rule on the elements of a subset of the set of natural numbers.	I	- This function rule may be, for example: 

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Point out the patterns in certain situations. Examples: [even number] + 2 = [even number] [any number] x 5 = [number ending with 0 or 5] [odd number] - 3 = [even number] etc. - a number is divisible by 2 if its last digit is an even number; - a number is divisible by 3 if the sum of the numbers associated with its digits is divisible by 3; - a number is divisible by 5 if its last digit is 0 or 5; - a number is divisible by 6 if it is divisible by 2 and by 3; - a number is divisible by 10 if its last digit is 0; - a number is divisible by 100 if its last two digits are 0.
1.2.7 TO FIND other numbers belonging to a sequence, given certain elements of the sequence.	I	- These are a few examples of sequences: 1, 2, 3, 5, 8, 13, 21, 34, 55, ... 0, 3, 6, 9, 12, 15, 18, 21, 24, ... 2, 5, 8, 11, 14, 17, 20, 23, 26, ... 1, 4, 9, 16, 25, 36, ... etc. - It is not necessary to formalize the correspondence rule and to state it in symbolic form. - This objective is intended to develop a certain ability to extrapolate. - The teacher can take the opportunity to make the pupil aware of the usefulness of a mathematical model, however rudimentary.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The calculator may be used to advantage as a tool for checking the accuracy of the correspondence rule. - ENRICHMENT: The teacher could present more complex sequences. Example: 1 , 3 , 4 , 6 , 7 , 9 , 10 , ... +2 +1 +2 +1 +2 +1 ...
1.2.8 TO DETERMINE some possible results obtained via a counting process, given concrete situations.	I	- Here is an example: An x2-type bacterium has a 5-hour life-span. If after an hour it produces one new bacterium, how many bacteria will there be after: - 3 hours? - 4 hours? - 5 hours? - 6 hours? Two hypotheses can be formed: - one bacterium dies without producing a new bacterium; - one bacterium dies immediately after having produced a new bacterium. The model shown here is based on the first hypothesis. It would be possible, however, to construct a model based on the second hypothesis. Legend: • represents a living bacterium; ◦ represents a dead bacterium.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
1.2.9 TO FIND the number of possible results using a Cartesian product, a tree diagram, etc., given concrete situations.	I	- This type of situation provides the opportunity to use: - games, - riddles, - puzzles. It also promotes the development of "problem-solving" strategies. The teaching notes present different strategy-developing activities which can be used here. Here are two examples: 1. A motorist has to cross three intersections where he can turn either to the right or to the left. Indicate the number of places he can reach. Draw a graph to represent this situation. 2. Construct a tree diagram which enables you to select the 25-cent, 10-cent, and 5-cent coins in a vending machine. What do you think the intersections in the diagram mean?

TERMINAL OBJECTIVE 1.3 TO PERFORM basic operations on integers using various algorithms.

The study of integers has many varied concrete applications: debit or credit, temperature, altitude, location, etc. It also promotes the use of problem-solving strategies.

In a broad sense, there are four main stages in the problem-solving process:

- understanding the problem;
- developing a plan to solve the problem;
- executing the plan;
- analyzing the solution.

These stages involve a certain number of strategies which are all geared to the solution of the problem involved.

This is a partial but comprehensive list of strategies applicable to each of these stages.

1. Understanding the problem:
   - Read the problem several times and make sure that each word is understood;
   - Reformulate the problem in your own words;
   - Identify the data;
   - Identify what is being sought;
   - Illustrate the data.

2. Developing a plan to solve the problem:
   - Eliminate non-pertinent data;
   - Find the missing data;
   - Construct or complete a figure;
   - Describe the principal conditions which the result should meet;
   - Subdivide the problem into smaller problems;
   - List the criteria on which to base your choices in the context of a decision-making problem;
   - Find one or several statements, one or several formulas, one or several properties, etc.;
   - Formulate hypotheses;

- Do a simulation of the problem;
- Find a model;
- Backtrack;
- Proceed by elimination;
- Determine the appropriate operation(s) in a given order;
- Select the most suitable tools, procedures, algorithms;
- Make use of various available resources.

3. Executing the plan:

- Evaluate if the solution is possible;
- Perform the required calculations;
- Associate the problem with a concept, a formula, an algorithm;
- Use a table, a calculator, etc.

4. Analyzing the solution:

- Check the correctness of the results;
- Check the calculations and the procedure followed;
- Look for other possible procedures;
- Provide complete information in the answer;
- Create and solve similar problems.

When applied to the solution of written problems, the strategies described above could be summed up in the following way:

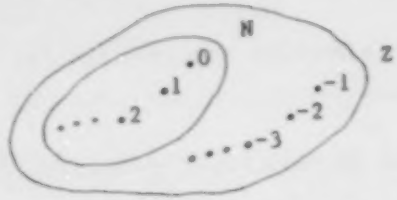
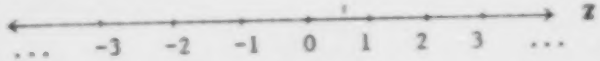
1. Read the problem carefully;
2. Restate the problem in your own words;
3. Select the pertinent data in the problem;
4. "Mathematize" the situation;
5. Estimate the result;
6. Solve the problem;
7. Interpret the result obtained.

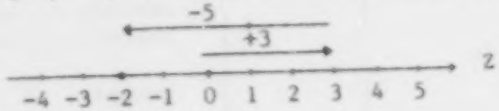
Here is an example:

> An escalator with 90 steps goes up at a rate of 2 steps per second. If a pupil tries to go down at a speed of 3 steps per second, how much time will it take him to arrive at the bottom of the escalator?

Solving such a problem requires the use of a number of strategies drawn from the preceding list.

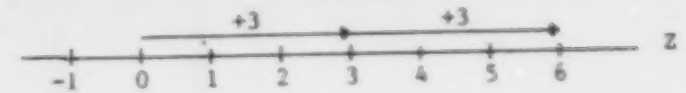
Pupils should be encouraged to use the calculator to check the accuracy of the result of operations performed on integers. However, they should not be dependent upon this instrument. It is essential that they master function rules, while taking into account the restrictions mentioned in the notes on the following pages.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
1.3.1 TO COMPARE two integers.	I	- The entire class of integers can be presented as an extension of the entire class of natural numbers. The Venn diagram below illustrates this idea.  - The set of integers can equally be represented by: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ or $\mathbb{Z} = \{0, -1, +1, -2, +2, -3, +3, \ldots\}$ and on the number line of integers, it will appear like this:  - It is understood that $\mathbb{Z}^* = \{-1, +1, -2, +2, -3, +3, \ldots\}$. - It may be useful to connect the relationship between two integers with their representation on the number line of integers. - It would be advisable to use the symbols $<$, $=$, and $>$ to compare integers. - Use concrete situations which can be described by means of integers: - temperature - altitude and depth - debit and credit - time

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
1.3.2 TO CALCULATE the sum of two or more integers.	I	- Comparing integers is an activity which the pupil has already performed by the end of his elementary schooling: "To compare relative whole numbers using concrete material." - It is strongly recommended that the teacher introduce the pupil to the concept of opposites: • -7 as the opposite of 7; • 5 as the opposite of -5. - Introduce the pupil to different algorithms for adding integers. - Comparison is one of these algorithms: - Lucy weighs 15 kg less than Peter. - I have 150$ more than you have. - etc. - Representation on a number line also makes it possible to solve additions. (+3) + (-5) would be represented by:  - A beam scale: - The mass placed in the right dish is symbolized by a positive number; - The mass placed in the left dish is symbolized by a negative number; (-5) + (3) = -2

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		 The reading will be -2. - It is important to have the pupils discover the function rule for adding integers by using the calculator or concrete situations such as the computation of the balance between debits and credits. The pupil should master this function rule for additions. - If a and b are integers, a + b should be considered an integer. Example: -3 + 5 is an integer just as 2 is an integer.
1.3.3 TO CALCULATE the difference between two integers.	I	- If a and b are two integers, a - b can be defined as being equivalent to: a + (the opposite of b). - Point out to the pupil that a subtraction operation on two integers amounts to adding to the first the opposite of the second, and that the function rule for additions applies. Example: 7 - 15 = 7 + (-15) = -8. This technique will be used later for subtractions of polynomials. - If a and b are integers, a - b should be considered an integer. Example: - 3 - (+5) is an integer just as -8 is an integer.

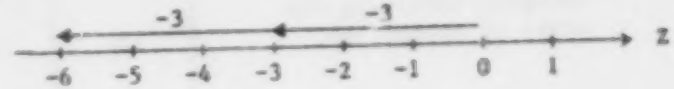
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
1.3.4 TO CALCULATE the product of two or more integers.	II	- Introduce pupils to different algorithms for multiplying integers. - Help pupils discover the function rule respecting the product of: - a positive integer by a natural number; - a negative integer by a natural number; Consider only these two cases, for in most real life situations the pupil will not have to multiply a quantity by a negative value. - Comparison can also be used to help pupils understand the product of integers: - This box is three times as heavy as that one; - His debt is twice as big as mine. - The beam-scale: - The mass placed in the right dish is symbolized by a positive number; - The mass placed in the left dish is symbolized by a negative number; $2 \times (-3) = -6$ Point out that 2 times (-3) amounts to adding (-3) + (-3), which is equal to -6.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
1.3.5 TO CALCULATE the quotient of two integers.	II	- Representation on a number line also makes it possible to solve products: 2 x 3 = 6  - If a and b are integers, ab should be considered an integer. Example: (-3)(-5) is an integer just as 15 is an integer. - ENRICHMENT: To introduce the function rule respecting the multiplication of integers. - Help pupils discover the function rule concerning the division of: - a positive integer by a positive natural number; - a negative integer by a positive natural number. - Here again, only these two cases should be considered, as in most everyday situations the pupil will not have to divide a quantity by a negative number. - Consider only cases where the quotient is "exact." - Point out to pupils the resemblance between this function rule and the rule used for multiplication. - Comparison can also be used to help pupils understand the quotient of integers: - The Lake of Two Mountains is half the depth of the Saint Lawrence River near Quebec City. - Sylvia has twice as much money as Dominique.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES

- Representation on a number line also makes it possible to solve quotients:

$-6 \div 2 = -3$



- The beam-scale:
  - to perform the operation $-6 \div 2$, simply remove half the mass from the left dish. The result -3 should be displayed.

- Find the function rules through patterns.

Examples:

$4 + 4 = 8$	$4 - 4 = 0$	$4 \times 4 = 16$	$4 \div 4 = 1$
$4 + 3 = 7$	$4 - 3 = 1$	$4 \times 3 = 12$	$4 \div 3 = \frac{4}{3}$
$4 + 2 = 6$	$4 - 2 = 2$	$4 \times 2 = 8$	$4 \div 2 = 2$
$4 + 1 = 5$	$4 - 1 = 3$	$4 \times 1 = 4$	$4 \div 1 = 4$
$4 + 0 = 4$	$4 - 0 = 4$	$4 \times 0 = 0$	$4 \div 0$
$4 + (-1) = 3$	$4 - (-1) = 5$	$4 \times (-1) = -4$	$4 \div (-1) = -4$
$4 + (-2) = 2$	$4 - (-2) = 6$	$4 \times (-2) = -8$	$4 \div (-2) = -2$
$4 + (-3) = 1$	$4 - (-3) = 7$	$4 \times (-3) = -12$	$4 \div (-3) = -\frac{4}{3}$
$4 + (-4) = 0$	$4 - (-4) = 8$	$4 \times (-4) = -16$	$4 \div (-4) = -1$
$4 + (-5) = -1$	$4 - (-5) = 9$	$4 \times (-5) = -20$	$4 \div (-5) = -\frac{4}{5}$
$4 + (-6) = -2$	$4 - (-6) = 10$	$4 \times (-6) = -24$	$4 \div (-6) = -\frac{4}{6}$

- ENTRICHMENT:
  - To introduce the function rule respecting the division of integers.
  - To bring out the similarity between this function rule and the rule used for multiplication.
  - To consider only cases where the quotient is "exact."

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
1.3.6 TO PERFORM operations on integers involving the order of operations.	II	- It is advisable to review briefly: - the order of operations; - the role of parentheses in modifying this order; - the distinction between term and factor. - The calculator could be used as an excellent means of checking the order of operations and the role of parentheses. - It is suggested that a single level of parentheses be used. - The calculator can be used for a quick check of the results. It also makes it possible to observe what effect changing one of the symbols of an expression +, −, x, ÷, ( ), has on that particular expression. Example: (3 + 5) x 4 equals 32 3 + 5 x 4 equals 23 - The use of the calculator should offer the opportunity to stress the necessity of entering such an expression accurately and of following strict writing rules in composing a mathematical expression. - It is strongly recommended that concrete situations be used when studying this objective: - successive moves; - binary operations; - temperature changes; - rotations of a wheel (number of rotations in integers); - etc. - ENRICHMENT: - Present complex expressions with possibly one or two levels of parentheses.

GENERAL OBJECTIVE

2

TO ENCOURAGE the pupil to apply his knowledge of rational numbers.

THEME: Rational numbers.

TIME: 27% in Secondary I

12% in Secondary II

In the first cycle of the secondary school program, the pupil is introduced to two different distinct forms of rational numbers. The program discusses fractional representation in which the fraction $\frac{a}{b}$ is defined as the quotient of two integers a and b, b not equal to zero. The program also covers decimal representation, a system which has countless applications, especially since the advent of the International System of Units (SI) and the development of the electronic calculator. In Secondary I the study of rational numbers will be limited to positive numbers, whereas in Secondary II the pupil will handle both positive and negative numbers.

Since the elementary school Mathematics Program does not consider the knowledge of the algorithms of operations on fractions a priority, these basic skills will be developed within the framework of the secondary school program. While the pupil's acquisition of operational algorithms is not neglected, the joint influence of the SI and the calculator makes it necessary to stress the study of decimal numbers.

However, it is not merely historical circumstances that justify the study of fractions. Indeed, fractions make it possible to mathematize a great number of concrete situations: half a person's age, 2/3 of a person's salary, 3/4 of a container, 9/5 of a segment, and so on. The use of ratios and percentages is also very frequent at a particular stage in the "mathematization" process.

For practical purposes the program emphasizes the decimal representation of rational numbers. There are many reasons why it stresses the development of the skills required to compare, to round off, to estimate and to perform all sorts of operations on decimal numbers. It is for the same reasons that the program explores this field of investigation in such detail and stresses situations which are favorable to the study of fractions and decimals. Among all these reasons, it is possible to mention the easier transition from percentage to decimal notation, greater speed in the performance of operations on decimal numbers with the calculator, the increasing use of the SI, and a greater ability to estimate a result using rounding off techniques.

TERMINAL OBJECTIVE 2.1 TO PERFORM basic operations on fractions using various algorithms.

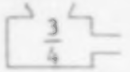
At this point the study of fractions should be limited to positive fractions, which are defined as the quotient of two natural numbers with a non-zero denominator. It is advisable to handle only simple fractions such as : $\frac{2}{5}$, $\frac{1}{2}$, $\frac{7}{10}$, $\frac{3}{4}$, $\frac{8}{7}$, and so on.

The pupil should be given familiar situations which he can understand. Consider, for example, the statement of the following problem:

> If one-third of a group of pupils plays tennis and two-fifths of the same group play ball, find the fraction of pupils who do not participate in either of these games, assuming that the games are played simultaneously. In addition, find the number of pupils in each of these three sub-groups if the group includes 30 pupils.

Certain aspects of this situation will be highlighted in the notes accompanying the intermediate objectives which follow.

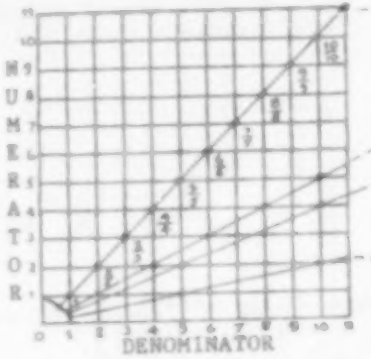
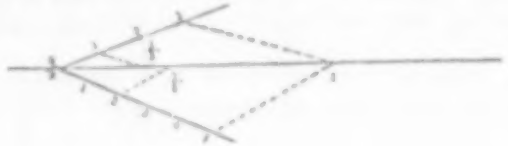
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.1.1 TO ASSOCIATE a fraction with its various meanings.	I	- Present the rational numbers as all the numbers expressed as $\frac{a}{b}$ in which a and b are integers and b is not equal to zero. - Point out to the pupil that the form $\frac{a}{b}$ is a symbolic representation of a rational number, and that later he will learn about another type of symbolic representation, namely, the decimal representation of a rational number. - It is important to point out to the pupil that division by zero is not possible. The calculator is an excellent means of verifying this "curious" behaviour of numbers. - The pupil has learned in elementary school to handle fractions using concrete materials. One of the terminal objectives presented to the pupil at that level was to discover the meaning of a fraction by means of various experiments. - There is another objective of the elementary school program: to perform additions, subtractions, multiplications on fractions using concrete materials, taking into account certain restrictions on numbers. - The concept of fraction can have many interpretations: (a) Fraction: quotient $\frac{3}{4}$ meaning "3 divided by 4" (b) Fraction: part of a whole $\frac{3}{4}$ meaning (c) Fraction: region $\frac{3}{4}$ meaning

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		(d) Fraction: ratio, $\frac{3}{4}$ meaning
		(e) Fraction: segment, $\frac{3}{4}$ meaning
		(f) Fraction: operator, 
		(g) Fraction: decimal, $\frac{3}{4}$ meaning $\frac{75}{100} = 0,75$
		(h) Fraction: repetition of fraction units, $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
		(i) Fraction: a point on the number line,
		(j) Fraction: measure, $\frac{3}{4}$ kg
		(k) Fraction: an ordered pair, three quarters: (3, 4).

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The pupil could be asked, as a supplementary activity, to collect from the newspapers a number of situations involving fractions and to interpret those situations.
2.1.2 TO REDUCE a fraction to its lowest terms.	1	- To reduce a fraction to its lowest terms (to simplify a fraction), it is possible to have the pupil find the greatest common divisor of the two terms and simplify using this common factor. Example: $\frac{20}{36} = \frac{4 \times 5}{4 \times 9} = \frac{\cancel{4}^{1} \times 5}{\cancel{4}_{1} \times 9} = \frac{5}{9}$ since $\frac{4 \times 5}{4 \times 9} = \frac{4}{4} \times \frac{5}{9} = 1 \times \frac{5}{9} = \frac{5}{9}$. - It is also possible to decompose each term into prime factors until the phase of reduction by elimination of the prime factors is reached. Example: $\frac{20}{36} = \frac{2 \times 10}{2 \times 18} = \frac{2 \times 2 \times 5}{2 \times 2 \times 9} = \frac{2}{2} \times \frac{2}{2} \times \frac{5}{9} = 1 \times 1 \times \frac{5}{9} = \frac{5}{9}$. - Another way of proceeding to reduce the fraction $\frac{20}{36}$ would be to reason thus: (1) 36 - 20 = 16, (2) the largest factor of 16 which also divides both 36 and 20 is 4, (3) therefore: $\frac{20}{36} = \frac{4 \times 5}{4 \times 9} = \frac{4}{4} \times \frac{5}{9} = 1 \times \frac{5}{9} = \frac{5}{9}$; or (1) 36 - 20 = 16 (2) 16 does not divide 20 (3) 16 - 20 = -4

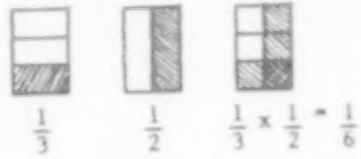
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		(4) 4 divides 20 (5) therefore 4 is the Greatest Common Divisor of 36 and 20. - The algorithm of the greatest common divisor should be studied only for use in reduction of fractions. Example: the number $\frac{20}{36}$ is equal to $\frac{20 \div 4}{36 \div 4}$ because the two terms of this fraction are divisible by 4, which is the greatest common divisor of 20 and 36. - To reduce a fraction, it is possible to determine the greatest common divisor by means of this often used algorithm: | | 20 | 36 | |---|---|---| | 2 | 10 | 18 | | x 2 | 5 | 9 | | 4 | | | To arrive at this result, it is a simple matter of dividing the two terms of the fraction by the prime numbers which are common divisors for these terms (or for the quotients), beginning with the lowest numbers. The process ends when it is no longer possible to find prime numbers which are common divisors for these two terms. - ENRICHMENT: - TO FIND the greatest common divisor of several natural numbers. Attainment of this objectve should eventually enable the pupil to use the concept of simple factoring.

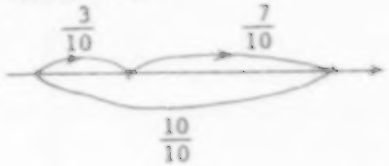
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.1.3 TO GENERATE fractions equivalent to a given fraction.	I	- The aim of this objective is to familiarize the pupil with the different ways of representing the same number. Example: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \ldots$ All these fractions are equivalent and represent the same rational number: $\frac{1}{2}$. These can be referred to as the equivalent set $\frac{1}{2}$. - The concept of equivalent fractions will be very useful for: - comparing fractions, - reducing fractions, - adding and subtracting fractions. - It is possible to generate equivalent fractions using the stability property of multiplication and division. Example 1: Fractions equivalent to $\frac{3}{5}$: $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{3 \times 3}{5 \times 3} = \frac{3 \times 4}{5 \times 4} = \frac{3 \times 5}{5 \times 5} = \ldots$ $\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25} = \ldots$ Example 2: Fractions equivalent to $\frac{12}{24}$: $\frac{12}{24} = \frac{6 \times 2}{12 \times 2} = \frac{4 \times 3}{8 \times 3} = \frac{3 \times 4}{6 \times 4} = \ldots$

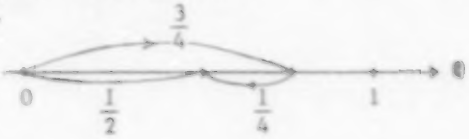
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		$\frac{12}{24} = \frac{6}{12} = \frac{4}{8} = \frac{3}{6} = \ldots$ - Partial representation of the sets of equivalent fractions by means of Cartesian coordinates can also be used to determine the element of the set closest to the origin, that is, the non-reducible equivalent fraction.  $\left\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6}, \frac{7}{7}, \frac{8}{8}, \_, \frac{10}{10}\ldots\right\}$ $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}, \frac{9}{18}\ldots\right\}$ $\left\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}, \frac{12}{30}, \frac{14}{35}, \frac{16}{40}\ldots\right\}$ $\left\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}, \frac{6}{30}, \frac{7}{35}, \frac{8}{40}\ldots\right\}$
2.1.4 TO COMPARE a set of fractions.	I	- In elementary school, the pupil has learned "to order fractions bearing in mind determined restrictions." - Recall the meaning of the symbols: $<$, $=$, and $>$. - One way of making this comparison is to locate these rational numbers on the rational number line. The relative position of these numbers will make it easier to compare them. (Cf. Teaching Notes.) To find out if the group of pupils playing tennis is larger or smaller than the group playing ball, this procedure can then be followed: 

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		Since $\frac{1}{3}$ is to the left of $\frac{2}{5}$, it can be said that $\frac{1}{3} < \frac{2}{5}$, and it is possible to conclude that there are fewer pupils playing tennis than there are pupils playing ball. - Another way of proceeding would be to find in the sets of equivalent fractions those that have the same denominator: $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \boxed{\frac{5}{15}} = \frac{6}{18} = \ldots$ $\frac{2}{5} = \frac{4}{10} = \boxed{\frac{6}{15}} = \frac{8}{20} = \ldots$ and to compare the numerators of these two fractions: $5 < 6$. Therefore: $\frac{5}{15} < \frac{6}{15}$ and: $\frac{1}{3} < \frac{2}{5}$ This technique is also valid if you are working with common denominators which are multiples of the least common denominator. - The algorithm of the least common multiple should be studied for use only when: - looking for fractions with a common denominator; - ordering fractions; - adding or subtracting fractions.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		Example: The fractions $\frac{5}{12}$ and $\frac{3}{10}$ are equivalent respectively to $\frac{25}{60}$ and $\frac{18}{60}$ if the least common multiple is used. - These are two often used algorithms: (a) | | 12 | 10 | |---|---|---| | 2 | 6 | 5 | | 2 | 3 | 5 | | 3 | 1 | 5 | | x 5 | 1 | 1 | | 60 | | | To arrive at this result it is a simple matter of dividing the denominators (or quotients) by their prime number divisors, beginning with the smallest prime numbers. The process ends when the number 1 appears at the bottom of each column. (b) Another way of proceeding is to multiply the denominators and to divide this product by the greatest common divisor of these two numbers. For example, the least common multiple of 12 and 15 is formed thus: $\frac{12 \times 15}{3} = \frac{180}{3} = 60.$ - No more than 5 fractions should be used.
2.1.5 TO CALCULATE the product of two or more fractions.	I	- In elementary school, the pupil learns "to perform multiplications of fractions whose numerator is 1, using concrete material." - Point out to the pupil that it is often very helpful to reduce the terms of the fractions before using the algorithm of multiplication.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The most common situations in everyday life are those in which a fraction is multiplied by an integer. Example 1: $150 \text{ km} \times \frac{7}{10} = 15 \text{ km} \times 7 = 105 \text{ km}$. Example 2: If there are 30 pupils in a group, the number of pupils playing tennis can be found this way: $30 \text{ pupils} \times \frac{1}{3} = 10 \text{ pupils}$. Example 3: If there are 30 pupils in a group, the number of pupils playing ball can be found this way: $30 \text{ pupils} \times \frac{2}{5} = 6 \text{ pupils} \times 2 = 12 \text{ pupils}$. To find the product of two fractions, a rectangular region can be used for reasoning: $\frac{1}{3} \times \frac{1}{2}$  $\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ However, using the properties of operations makes it possible to proceed as follows: $\frac{5}{2} \times \frac{7}{5} \times \frac{11}{7} = \frac{5 \times 7 \times 11}{2 \times 5 \times 7} = \frac{\cancel{5}^{1} \times \cancel{7}^{1} \times 11}{\cancel{5}_{1} \times \cancel{7}_{1} \times 2} = \frac{11}{2}$.
2.1.6 TO CALCULATE the sum of two or more fractions.	I	- In elementary school, the pupil learns "to perform additions, etc., where the denominator of one of the fractions is a multiple of the other (or others), using concrete material." - The additions presented should gradually increase in complexity:

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		(1) Use fractions with the same denominator. Example: $\frac{3}{10} + \frac{7}{10} = \frac{10}{10} = 1$.  (2) Use fractions in which one of the denominators is a multiple of the other. Example: $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$. (3) Use fractions whose denominators are relatively prime. Example: $\frac{1}{3} + \frac{3}{4} = \frac{4}{12} + \frac{9}{12} = \frac{13}{12}$. (4) Use any kind of fractions. Example: $\frac{5}{12} + \frac{3}{8} = \frac{10}{24} + \frac{9}{24} = \frac{19}{24}$. - It is recommended that simple fractions which are equal to, smaller, or larger than the unit be used. - In the example given at the beginning, the fraction of pupils participating in a game is given by: $\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$.
2.1.7 TO CALCULATE the difference between two fractions.	I	- In elementary school, the pupil learns "to perform subtractions, etc., where the denominator of one of the fractions is a multiple of the other (or others), using concrete material."

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The subtractions presented should gradually increase in complexity: (1) Use fractions with the same denominator. Example: $\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$.  (2) Use fractions in which one of the denominators is a multiple of the other. Example: $\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$. (3) Use fractions whose denominators are prime factors. Example: $\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$. (4) Use any kind of fractions. Example: $\frac{7}{12} - \frac{1}{8} = \frac{14}{24} - \frac{3}{24} = \frac{11}{24}$. - Transformation into equivalent fractions is the appropriate algorithm here. Example: $\frac{3}{5} - \frac{1}{2} = \frac{6}{10} - \frac{5}{10} = \frac{1}{10}$. - Point out that subtraction is not always possible with all the positive rational numbers. - It is recommended that simple fractions be used.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- In the example given at the beginning the fraction of pupils not participating in any game is given by: $1 - \frac{11}{15} = \frac{15}{15} - \frac{11}{15} = \frac{4}{15}$.
2.1.8 TO CALCULATE the quotient of two fractions.	I	- In elementary school the pupil has not performed divisions on rational numbers in the form $\frac{a}{b}$.
		- Teachers should confine themselves to the written form $\frac{a}{b} \div \frac{c}{d}$ in which b, c, and d are not equal to zero. The written form of complex fractions should not be used.
		- If the common denominator approach is taken, the operation becomes a simple matter of dividing the numerators. Example: $\frac{1}{2} \div \frac{1}{3} = \frac{3}{6} \div \frac{2}{6} = 3 \div 2 = \frac{3}{2}$.
		- This particular approach does not imply that the algorithm is defined as deriving from the fact that division is the inverse operation of the multiplication $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$ in which b, c, and d are not equal to zero.
		- It is recommended that simple fractions be used.
		- Some concrete situations can be interpreted by means of division. Example: How many containers are needed to hold four litres of paint if each full container holds $\frac{2}{3}$ of a litre? $4 \div \frac{2}{3} = 6$. 6 containers are needed.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.1.9 TO ESTABLISH a ratio which represents a real-life situation.	I	- This objective aims to teach the pupil to use the concept of fraction to interpret certain real-life situations. Examples: If you travel 100 km and the total distance is 150 km, what is the ratio of the distance covered to the total distance? If there are 5 boys and 8 girls in a classroom, what does the ratio $\frac{5}{13}$ represent? - Through this objective the pupil takes a new step toward the "mathematization" of real situations.

TERMINAL OBJECTIVE 2.2 TO SOLVE problems representing real-life situations involving decimal numbers.

The use of decimal numbers is increasingly widespread, and it is essential for the pupil to know them well in order to understand and interpret the various situations he will have to deal with.

In addition to developing the pupil's competence with the different algorithms of the basic operations on decimal numbers, the program puts particular emphasis on the development of such skills as comparing, rounding off, and estimating. Here the calculator should be used only to check the results so as to force the pupil to master the basic algorithms of the operations.

The teacher is strongly advised to start with concrete situations and to stress the "problem-solving" approach as described in Terminal Objective 1.3.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.2.1 TO READ decimal numbers.	I	- In elementary school the limit of this objective is the hundredths place. - It is suggested that the limit be extend to the thousandths place. - Use concrete situations to present the numbers to be read. Example: John is 1,80 m tall. - The number 4,2 can be read as follows: "four decimal two" or "four and two-tenths." - ENRICHMENT: The limit could be the hundred-thousandths.
2.2.2 TO WRITE decimal numbers.	I	- Present the rules respecting the writing of numbers: (a) Using digits: Insist on the correct application of the writing rules presented in Document #16-3136A, International System of Units at the Secondary Level, D.G.D.P., 1977. (b) Using letters: Review the grammar rules respecting the writing of numbers. - Applications: Writing a given amount on: - a personal cheque, - a receipt, - a bill, - a contract, - a lease.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.2.3 TO GRAPH a decimal number on the number line.	I	- This objective makes it possible to place a decimal number in the right position within an interval on the number line. Example: It can be said that 3,55 is located approximately at point a.
2.2.4 TO COMPARE decimal numbers and fractions.	I	- In elementary school the pupil learns "to order decimal numbers up to the hundredths place." - Review the meaning of the symbols $<$, $=$, and $>$. - These comparisons should be made by means of concrete situations involving: - masses, - prices, - lengths, - etc. - When the comparison is between a decimal number and a fraction, the technique to be used depends on the situation. Example 1: Compare $\frac{3}{4}$ and 0,72. - $\frac{3}{4}$ transformed into a decimal number is 0,75. - So, $0,75 > 0,72$. - Therefore, $\frac{3}{4} > 0,72$. Example 2: Compare $\frac{2}{3}$ and 0,6. - 0,6 is transformed into a fractional number: $\frac{6}{10} = \frac{3}{5}$. - Equivalent fractions with the same denominator are found: $\frac{2 \times 5}{3 \times 5} > \frac{3 \times 3}{5 \times 3} \quad \therefore \frac{10}{15} > \frac{9}{15} \quad \therefore \frac{2}{3} > 0,6.$

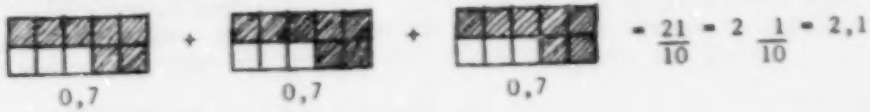
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.2.5 TO DETERMINE the place value of a digit in a decimal number.	I	- The pupil should be able to compare numbers up to the thousandths place. Example: I have two metal blocks A and B weighing respectively $\frac{2}{3}$ kg and 0,435 kg. Which of the two pieces is heavier? Answer: Block B, because $\frac{2}{3} > 0{,}435$. - Review the distinction between "digit" and "number." - Consider only cases in which numbers are expressed in base ten. Example: Determine the value of 2 in the number 29,78. Answer: Its value is 20 units (ones). Its value is 2 tens. - Certain concrete situations can help the pupil to attain this objective. Example: How many hundreds of dollars are there in \$875,43? Answer: There are 8.
2.2.6 TO DETERMINE the number of units, of tens, of tenths, of hundreds, of hundredths, etc., in a decimal number.	I	- This objective is a generalization of the preceding objective. Example: In the number 1 345,27 there are: 134 527 hundredths 13 452 tenths 1 345 units (ones) 134 tens 13 hundreds 1 unit of thousands 0 tens of thousands.

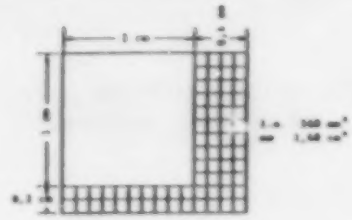
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.2.7 TO WRITE a decimal number in expanded form and vice-versa.	I	- Certain situations can also be interpreted by means of this concept. Example: How many dimes are there in $123,42? Answer: There are 1234 dimes. - This objective aims to enable the pupil to express the number 123,47 as follows: $123,47 = (1 \times 100) + (2 \times 10) + (3 \times 1) + (4 \times 0,1) + (7 \times 0,01)$ $= (1 \times 100) + (2 \times 10) + (3 \times 1) + (4 \times \frac{1}{10}) + (7 \times \frac{1}{100})$. - The pupil should be able to associate: $1 \times 100 + 3 \times 10 + 7 \times 1 + 2 \times 0.1 + 5 \times 0,01$ with 137,25.
2.2.8 TO ROUND a decimal number to a given degree of accuracy.	I	- This objective makes it possible to generalize the application of Objective 1.1.7. - The pupil should be able to round off a number; - to the nearest hundred; - to the nearest ten; - to the nearest unit; - to the nearest tenth; - to the nearest hundredth. - For further details on the rules respecting the rounding off of numbers please see Appendix D in Document #16-3136A, International System of Units at the Secondary Level, D.G.D.P., 1977.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.2.9 TO EXPRESS a given fraction in decimal form and vice-versa.	I	- This objective has become much more important since the invention of the pocket calculator: - The number 12,345 678 displayed on the calculator can be rounded off to 12,35 (if it is done to the nearest hundredth); - To estimate the product 216 x 49. It can be rounded to 200 x 50, and the answer will be approximately 10 000. - The transition from fractional notation to decimal notation is made through division. The pocket calculator can be very helpful here. Example: $\frac{3}{7}$ = 0,428 571 42 ..., but if an answer rounded off to the nearest thousandth is needed, it should be written as follows: $\frac{3}{7} \approx 0{,}429.$ - In situations where a decimal number has to be presented in fraction form, only decimal numbers rounded off to the nearest thousandth should be used. Example: 0,852 gives $\frac{852}{1000} = \frac{213}{250}.$ - ENRICHMENT: - The use of the pocket calculator makes it possible to bring out certain patterns of numbers. - Periodicity is one of the interesting patterns to analyze: $\frac{1}{3} = 0{,}333\ 333\ \ldots$ etc.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- It is important to use an appropriate selection of fractions so as to bring out the period on the display screen. - This is the symbolism to use: $\frac{1}{2} = 0,500 \ldots = 0,5$ $\frac{1}{3} = 0,333 \ldots = 0,\overline{3}$ $\frac{1}{3} \neq 0,333.$ - It is important for the pupil to acquire a certain mathematical precision. He should be required to write: $\frac{1}{3} \simeq 0,333$ and not $\frac{1}{3} = 0,333.$ = means "... is equal to ..." $\simeq$ means "... is approximately equal to ..."
2.2.10 TO MULTIPLY quickly decimal numbers by powers of ten.	I	- With the introduction of the International System of Units, it has become necessary for the pupil to develop as much as possible the ability to multiply or divide mentally large and small numbers by ten. - It is advisable to help the pupil develop this ability by having him perform these operations on numbers outside the context of familiar situations to avoid the interference which could be caused by the use of SI units. These units will be studied later (Cf. General Objective 5). - Do not go beyond 10^{-4} for the small numbers. - ENRICHMENT: - TO EXPRESS very small or very large numbers in scientific notation, using real-life situations.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		Here are a few examples: - The speed of light in space: 300 000 000 m/s 3,8 E 8 m/s - The distance between the Earth and the Moon: 353 680 km 3,54 E 5 km - The diameter of the Earth at the equator: 12 756 km 1,28 E 4 km - The diameter of the Earth at the poles: 12 713 km 1,27 E 4 km - One second in relation to one hour: $\frac{1}{3600}$ h 2,778 E -4 h - etc.
2.2.11 TO CALCULATE the sum of decimal numbers.	I	- In elementary school the pupil performs this operation on numbers not exceeding the hundredths place. - The pupil should be able to perform additions on a series of decimal numbers rounded off to the nearest thousandth. - Remind the pupil of the function rule for addition of integers and generalize its application to decimal numbers. Example: Cécile has received 98,37$ and 104,00$. How much does she have left after spending 75,52$ and 119,98$?
2.2.12 TO CALCULATE the difference between two decimal numbers.	I	- In elementary school the pupil performs this operation on numbers not exceeding the hundredths' place.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.2.13 TO CALCULATE the product of decimal numbers.	I	- The pupil should be able to perform a subtraction involving decimal numbers rounded off to the nearest thousandth. Example: Paul pays a deposit of 32,48$ on a bicycle with a purchase price of 129,35$ (tax included). How much money does he owe? - In elementary school the pupil has learned "to multiply two decimal numbers the product of which does not exceed the hundredths place." - The pupil should be able to perform multiplications on a series of decimal numbers, the product of which is rounded off to the nearest ten-thousandth. - Use only positive decimal numbers. - These are two situations which make it possible to visualize the product: (a) Compute: 0,7 x 3  $= \frac{21}{10} = 2\ \frac{1}{10} = 2,1$ or: $3 \times 0,7 = 3 \times \frac{7}{10}$ $= \frac{21}{10}$ $= 2\ \frac{1}{10}$ $= 2,1$ or:

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		(b) Perform: 1,4 x 1,2 1,4 x 1,2 = 1,68 A 1 cm^2 square An 8 mm^2 rectangle A 20 mm^2 rectangle and a 40 mm^2 rectangle. 
2.2.14 TO CALCULATE the quotient of two decimal numbers.	I	- In elementary school the pupil has learned "to divide a decimal number by a positive whole number less than 10." - The pupil should be able to perform divisions on a series of decimal numbers in which the quotient is rounded off to the nearest ten-thousandth. - The average pupil experiences difficulties when the dividend, the divisor, or the quotient contain a number of 0's or 9's. It is advisable to present a number of cases in which this situation occurs. - Use only positive decimal numbers. Example: Finding how many bookshelves 1,2 m in length it is possible to build from a 5,35 m long board is a problem involving division of decimal numbers: 5,35 ÷ 1,2. The equivalence of numbers makes it possible to present this division as being similar to the division of natural numbers. 5,35 ÷ 1,2 = 5,35 ÷ 1,20 = $\boxed{5,35 \times 100}$ $\boxed{1,20 \times 100}$ = 535 ÷ 120. Note that in this situation an answer such as (4) should be accepted as appropriate. Take this opportunity to start a discussion on the necessity of pursuing the division.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.2.15 TO SOLVE problems requiring fractions and decimal numbers.	II	- It is strongly recommended that the function rules respecting integers be reviewed briefly and their applications to decimal numbers be extended. The same limits as those on integers should be respected: - multiplying a decimal number by a positive decimal number; - dividing a decimal number by a strictly positive number; - Remind the pupil of certain problem-solving strategies using realistic situations. Example 1: A merchant sells grain at 12,48$ for 3 kg and another merchant charges 8,50$ for 2,25 kg. Which purchase price is more advantageous? Answer: 8,50$ for 2,25 kg is the better price. Example 2: The distance between Québec and Montréal is approximately 256 km. At two thirds of the distance a motorist who left with a full tank of gasoline required 24 litres to refill it. How many kilometres per litre does the car get? Answer: Approximately 7,1 kilometres per litre.
2.2.16 TO EXPRESS a given decimal number as a percent and vice-versa, given a realistic context.	I	- In an expression in the form n%, the symbol % means that the value n represents the first term of a ratio whose second term is 100. Example: 17% is an expression equivalent to $\frac{17}{100}$. Note that: $0\% = \frac{0}{100} = 0,00$ $17\% = \frac{17}{100} = 0,17$ $100\% = \frac{100}{100} = 1,00.$ For each percent between 0% and 100% there is a corresponding number between 0 and 1.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.2.17 TO EXPRESS a given fraction as a percent and vice-versa, given a realistic context.	I	- As a consumer the pupil will have frequent opportunities to use this concept. It is therefore important to ensure that it is well understood and to help the pupil acquire the operational skills which will enable him to handle the concept effectively. - Note: the symbol % does not represent an SI unit. - The decimal number 0,172, for instance, will be written 17,2%. Example: If 18 out of 25 pupils practice swimming, indicate the percentage of those who do not practice that sport. Answer: $\frac{7}{25} = 0{,}28 = 28\%$, therefore 28% of the pupils do not practice swimming. - It is strongly recommended that the teacher make the pupil aware that 100% corresponds to the unit and that consequently any percent between 0 and 100% is associated with a fraction between 0 and 1. Example: 15% can be associated with $\frac{15}{100}$ or $\frac{3}{20}$. - To express the fraction $\frac{1}{7}$ in percent, it is preferable to write 14,29% as an approximation of 14,285 7...% instead of the expression $14\ \frac{2}{7}\%$. - However, it is not recommended that fractions of this type be used because they are rarely dealt with in real situations. Instead, use such fractions as: $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{5}$, etc. - Another procedure for finding the percent corresponding to a given fraction is to use equivalent fractions. Examples: (a) $\frac{3}{4} = \frac{3}{4} \times \frac{25}{25} = \frac{75}{100}$, which gives 75%. (b) $\frac{5}{8} = \frac{5}{8} \times \frac{25}{25} = \frac{125}{200} = \frac{125 \div 2}{200 \div 2} = \frac{62{,}5}{100}$, which gives 62,5%.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The rule of three can also be used. Examples: (a) $4 \longrightarrow 100$ $3 \longrightarrow x$ therefore: $x = \frac{100 \times 3}{4} = \frac{300}{4} = 75$, which gives 75%. (b) $8 \longrightarrow 100$ $5 \longrightarrow x$ therefore: $x = \frac{100 \times 5}{8} = \frac{500}{8} = 62,5$, which gives 62,5%.
2.2.18 TO EVALUATE mentally the quantity corresponding to a percentage, given a realistic situation.	I	- The aim of this objective is to facilitate the development of the reflex which makes it possible to evaluate very quickly the value of a tax (8%) or of a reduction (5%, 10%, 15%, 20%, 25%, 30%, 40%, and 50%). - A pupil may sometimes have to leave a tip. It would be very useful for him to be able to evaluate mentally the appropriate amount. Example: If a meal costs 17,64$, how much will the tip be? (a) If the tip is 10%: 17,64$ ⟶ 18,00$ ⟶ 1,80$. (b) If the tip is 15%: 17,64$ ⟶ 18,00$ ⟶ 1,80$ + 0,90$ ⟶ 2,70$, because: $15\% = 10\% + 5\% = 10\% + \frac{10\%}{2}$, therefore: 10% of 18,00$ gives 1,80$ $\frac{1}{2}$ of 1,80$ gives 0,90$ ⟹ 2,70$.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- It is important that the pupil be able to make an approximate evaluation of the correct amount of tax so as to be capable of detecting an error on the part of the salesperson. Example: The pupil should know that the salesperson has made a mistake by charging a tax amount of 1,50$ on a sales price of 12,50$ if the tax rate is 9%. - There are several ways of evaluating the amount of tax. Say we have an article which costs 12,50$. (a) - We round off 12,50$ to 13,00$; - We compute mentally: $\frac{13 \times 9}{100}$ = 1,17$; - The tax should not exceed 1,17$. (b) - We raise the tax from 9% to 10%; - We compute mentally 12,50 x $\frac{1}{10}$ = 1,25$; - The tax should not exceed 1,25$. (c) It is possible to obtain a better approximation by applying the following property: $9\% = 10\% - 1\% = 10\% - \frac{10\%}{10}$; then: • we compute: 12,50$ x $\frac{1}{10}$ = 1,25$, • then: 1,25$ ÷ 10 = 0,12 ⟹ 1,13$. In this case the answer is exact. This technique makes it possible to obtain a good approximation in situations where the numbers are more difficult to handle.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		Example: Evaluate the 8% tax on 352,65$. - Round off to: 350,00$ - Compute : 350,00$ x $\frac{1}{10}$ = 35,00$ - Then : -35,00$ ÷ 5 = -7,00$ ⟹ 28,00$ - The tax comes to about 28,00$.
2.2.19 TO CALCULATE a certain percent of a number, given a realistic context.	II	- Calculating the n% of a number simply involves multiplying this number by $\frac{n}{100}$ or by the corresponding decimal number. Thus, calculating 14% of 240 will be expressed as follows: 240 x $\frac{14}{100}$ or 240 x 0,14. However, it is acceptable for the pupil to write $\frac{14}{100}$ x 240 or 0,14 x 240 because it has been established that multiplication is a commutative operation. - The pupil should develop the ability to calculate the n% of a number because he will need this skill for: - evaluating a sales tax; - evaluating a discount; - calculating the interest on a loan or an investment; - etc. - Calculating a tax is an application of percent operations. The pupil should be able to use the algorithm which he can handle best on the operational level. He can proceed as follows: (a) 352,65$ x 0,08 ≈ 28,22$ (b) 352,65$ x $\frac{8}{100}$ ≈ 28,22$

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		The exact answer is 28,212$. The pupil has to round this number off to the higher hundredth. Example: In a town of 20 000 inhabitants, 51% of the population are over 18 years old. If 35% of the adult population (people over 18 years of age) smoke, how many smokers are there in the town? Answer: There are about 3570 smokers in the town.

TERMINAL OBJECTIVE 2.3 TO SOLVE problems dealing with buying or selling, using the concept of percentage.

This objective aims to help the pupil become more independent and a better prepared consumer. Thanks to the concept of percentage, mathematics can help the pupil to make more intelligent decisions with respect to the advantages and disadvantages in sales and purchase transactions.

If the pupil has a certain competence in calculations involving percentages, he should be allowed to use the calculator to solve a wider variety of problems.

Teachers should encourage the pupil to adopt an approach based on problem-solving strategies and should take the opportunity to promote the development of heuristics and strategies.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.3.1 TO CALCULATE the final price of an article, taking into account various factors such as the marked price, the reduction, the sales tax, etc.	II	- This objective offers ample opportunity for many applications of the concept of percentage. Teachers should select realistic situations which are consistent with the pupil's experience. - If the calculator is used, more complex examples or situations can be presented. Example: A store is offering a 15% discount on a bicycle with a price tag of 139,95$. Calculate how much that bicycle will cost if the provincial tax is 8%. Answer: The bicycle will cost 128,48$.
2.3.2 TO PREPARE a bill, given the number of articles bought, the price of each article, and the rate of tax.	II	- The best way to understand a bill and the pertinence of its components is to make one out. - As a sensible consumer, the pupil should learn to check his bills. Example: Make out an order form or a bill for a catalogue purchase.
2.3.3 In a purchase or sale context, given an amount and the percent to which it corresponds, TO CALCULATE 100% of this amount.	II	- It is important to present realistic situations Example: Find the sales price of a bicycle if it cost its owner $150,00, which includes an 8% tax. Here, the number given is 150 ($) and what it corresponds to in percent is 108 (%). The sales price of the bicycle corresponds to 100%. - If the calculator is used, more complex examples or situations can be presented.

TERMINAL OBJECTIVE 2.4 TO SOLVE problems involving proportions.

Proportions are widely used and they appear in numerous situations: enlargement or reduction of geometric figures, reading of scale maps, use of unit prices, percentage, etc. In fact, all these situations are based on the equivalence of ratios and on the properties of proportions which derive from them. In this context, the simple rule of three seems to be a special case of the properties of proportions.

To find the answer to the question, "Which is the more advantageous purchase, 5 kg of sugar at 3,59$ or 0,25 kg at 0,20$," the concept of proportion can be used. To reduce a 5 cm x 10 cm photograph to 75% of its original size, it is also possible to use proportions.

It is strongly recommended that extensive use be made of situations where proportions can be employed. In this context the calculator can be profitably used to develop certain problem-solving strategies.

Ratios are widely used. In geography, scales are used to indicate map reduction, 1:100 000, for instance. It is a scale based on a 1/100 000 ratio. If the basic unit on the map is 1 cm, that means that 1 cm on the map represents 1 kilometre in reality. Engineers and architects use the same symbolism to make their plans understood. In science, we write 1 cm = 1 m to refer to a ratio and it means: "1 centimetre corresponds to 1 metre." Here, it is compulsory to write the units because they are different. Additional information can be found in the document International System of Units at the Secondary Level (Code: 16-3136A).

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.4.1 TO DETERMINE ratios equivalent to a given ratio.	II	- The identity property of multiplication makes it possible to establish a sequence of equivalent ratios. Example: $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{3 \times 3}{4 \times 3} = \frac{3 \times 4}{4 \times 4} = \ldots$ $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \ldots$ - Similarly, the property of stability of division makes it possible to establish a sequence of equivalent ratios: $\frac{12}{20} = \frac{12 \div 4}{20 \div 4} = \frac{12 \div 2}{20 \div 2} = \frac{12 \div 0,5}{20 \div 0,5} = \ldots$ $\frac{12}{20} = \frac{3}{5} = \frac{6}{10} = \frac{24}{40} = \ldots$ Example: The recipe for a cake calls for 0.5 cups of sugar and 3 cups of flour. Find the number of cups of sugar for 3, 6, 9, 12, or 15 cups of flour.
2.4.2 TO DETERMINE whether two given ratios form a true proportion.	II	- The reference set used here is still the set of rational numbers. - <u>Definitions</u>: - Two ratios $\frac{a}{b}$ and $\frac{c}{d}$ form a true proportion if and only if they are equivalent. - If $\frac{a}{b}$ and $\frac{c}{d}$ are two ratios ($b \neq 0$ and $d \neq 0$), they form a true proportion if and only if $a \cdot d = b \cdot c$, and they can be written as follows: $\frac{a}{b} = \frac{c}{d}$.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The concept of proportion which is covered here does not need a long theoretical presentation. The subject will be studied in greater detail in the second cycle. - Vocabulary: in $\frac{a}{b} = \frac{c}{d}$, the letters a and d are called extremes whereas b and c are known as the means.
2.4.3 TO FIND the value of a missing term in a proportion.	II	- It is important to avoid using a specialized vocabulary so as not to make things too complicated for the pupil. Examples: (a) $\frac{2}{5} = \frac{x}{10}$. (b) $\frac{x}{7} = \frac{3}{11}$. (c) The proportion $\frac{2}{x} = \frac{x}{8}$, which gives $x^2 = 16$ and then $x = 4$, is one of a number of ways of introducing square root. Since the situations to be analyzed must be drawn from everyday life, $x = -4$ as the solution of $x^2 = 16$ will be overlooked. This second root of $x^2 = 16$ will be introduced in the second cycle of secondary school.
2.4.4 TO SOLVE a problem involving a simple proportion, given a realistic context.	II	- Solving a problem with the simple rule of three involves looking for the missing term of a proportion. - Here it is important to review briefly the main stages of the process to follow when solving a problem: 1. Read the problem carefully. 2. Restate the problem in your own words. 3. Select the important data of the problem. 4. "Mathematize" the situation (create the mathematical model).

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		5. Estimate the result. 6. Solve the problem. 7. Interpret the result. - The pupil will know how to apply the simple rule of three if the process he uses is well structured and if he clearly identifies the elements of the problem. Example: If 5 articles cost 7,00$, find the cost of 8 articles. It is possible to form the proportion: $\frac{5 \text{ articles}}{7.00\$} = \frac{8 \text{ articles}}{x\$}$ or: $\frac{5 \text{ articles}}{8 \text{ articles}} = \frac{7.00\$}{x\$}$ Both situations are acceptable, but it is recommended that the second proportion be used because the ratios are made of elements of the same type. - It would be advisable to present to the pupil a few situations similar to this one: If it takes 4 workers 20 hours to complete a job, find how many hours it will take 8 workers to complete a similar job. The answer the pupil finds may be unexpected. This is the best opportunity to make him think about how the result obtained should be interpreted, and to encourage him to look for other strategies likely to lead to the "right answer." (The answer here is obviously 10 hours.) In this case the best strategy is to return to the unit: How much time will it take one worker to complete the same job?

TERMINAL OBJECTIVE 2.5 TO FIND the square root of a number in certain situations.

There are several ways of determining the square root of a number:

- by estimation,
- by change to successive approximations,
- by using the square root table,
- by measuring in geometric constructions,
- by a method for extracting the square root,
- etc.

However, it is not part of the objectives of the first cycle program to use the fourth and the fifth procedures listed above, since geometric constructions are long and lacking in precision, and the use of a method of extraction can be quickly forgotten.

Nevertheless, the development of skills or techniques such as estimation, change to successive approximation, reading, etc. should be encouraged.

Square root should be used for, among other things:

- finding the missing term in a proportion in the form $\frac{2}{x} = \frac{x}{4}$;
- finding the measure of the side of a square whose area is known.

Extraction of the square root should be done in concrete situations:

If your neighbour has 150 square tiles 0,2 m in length, find the maximum length of the side of a square patio he can pave without breaking any tiles.

Anwer: The patio will be 2,4 m in length.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.5.1 TO CALCULATE the square of a given number.	II	- Memorizing certain natural numbers and their respective squares will enable the pupil to be better equipped to find by estimation the square root of a number. - This is a partial table of the principal numbers and their respective squares:

n	n^2	n	n^2	n	n^2
0	0	9	81	30	900
1	1	10	100	40	1 600
2	4	11	121	50	2 500
3	9	12	144	60	3 600
4	16	13	169	70	4 900
5	25	14	196	80	6 400
6	36	15	225	90	8 100
7	49	20	400	100	10 000
8	64	25	625		

- Here are a few examples which implement this objective:

(a) $5 \times 5 = 25$
(b) $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$
(c) $0,25 \times 0,25 = 0,062\ 5$
(d) $2,3 \times 2,3 = 5,29$.

2.5.2 TO IDENTIFY the number which is the square of a given number.	II	- The numbers given must be perfect squares. A number is a perfect square if its square root is a natural number. These perfect squares are located on the diagonal of a multiplication table.

X	0	1	2	3	4	5	. . .
0	[0]	0	0	0	0	0	. . .
1	0	[1]	2	3	4	5	. . .
2	0	2	[4]	6	8	10	. . .
3	0	3	6	[9]	12	15	. . .
4	0	4	8	12	[16]	20	. . .
5	0	5	10	15	20	[25]	. . .
.	.	.	.	.	.	.	. . .
.	.	.	.	.	.	.	. . .
.	.	.	.	.	.	.	. . .

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
2.5.3 TO FIND the square root of a fraction whose terms are the squares of natural numbers.	II	- Point out to the pupil that, to find the square root of a fraction, he should look for the square roots of the numerator and the denominator. Examples: (a) $\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2} = 0,5$ (b) $\sqrt{\frac{49}{36}} = \frac{\sqrt{49}}{\sqrt{36}} = \frac{7}{6}$ (c) $\sqrt{\frac{625}{100}} = \frac{\sqrt{625}}{\sqrt{100}} = \frac{25}{10} = 2,5.$
2.5.4 TO FIND the square root of a given number to a given degree of accuracy.	II	- The different ways of finding square roots are: - mental calculation, - the use of a table of squares, - the use of the calculator. - The degree of accuracy should not go beyond the nearest number if the calculation is done mentally or quickly. - Here is a series of examples in which the technique developed in Objective 2.5.3 is used: (a) $\sqrt{1,44} = \sqrt{\frac{144}{100}} = \frac{12}{10} = 1,2$ (b) $\sqrt{0,144} = \sqrt{\frac{144}{1000}} \simeq \frac{12}{30} = 0,4$ (c) $\sqrt{0,04} = \sqrt{\frac{4}{100}} = \frac{\sqrt{4}}{\sqrt{100}} = \frac{2}{10} = \frac{1}{5} = 0,2.$ - This objective aims at enabling the pupil to form some idea of the degree of accuracy of the square root of a number. Perfect squares should be used for this purpose.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		Example: Find the square root of 180. According to the tables, 180 is between 169 and 196: $169 < 180 < 196$. So: $\sqrt{169} < \sqrt{180} < \sqrt{196}$, therefore $13 < \sqrt{180} < 14$. It can be concluded that the square root of 180 is between the numbers 13 and 14. This technique is known as "successive approximations." - The list of the principal perfect squares to remember is found in the table of squares (Note 2.5.1) - The calculator can be used to check the results. - For a more accurate evaluation of the answer, it is recommended that the pocket calculator be used systematically. It will then be possible to help the pupil understand that he can find the square root of any positive number. Except when looking for the square root of perfect squares, the use of the calculator will make it clear that: $\sqrt{X}$ gives Y, and that: Y x Y does not give back X. The operation should be written: $Y \times Y \simeq X$, therefore $\sqrt{X} \simeq Y$ because the calculator truncates the real number, keeping only a certain number of significant figures.

GENERAL OBJECTIVE

3

TO PROMOTE the learning of some basic concepts of algebra.

THEME: Equations and inequations.

TIME: 15% in Secondary II

Before the pupil learns to solve equations or inequations properly, it is important to introduce him to the basic concepts and operations of algebra. The purpose is to have him find equivalent expressions, the numerical value of such expressions, and the values of given algebraic expressions.

Only in a problem-solving context does the acquisition and mastery of the techniques for solving equations and inequations make sense. Indeed, what is the use of knowing how to solve equations or inequations if the pupil never handles them in real situations?

It seems important, therefore, for the study of this objective to develop problem-solving heuristics. These will enable the pupil to construct a mathematical model when looking for an equation or an inequation, and to use the model to solve that situation before going on to interpret the solution which has been found.

In general, the problem-solving process comprises four main stages:

- Understanding the problem;
- Developing a plan for solving the problem;
- Implementing the plan;
- Analyzing the solution.

Here is an example:

> An escalator with 90 steps goes up at a speed of 2 steps per second. If a pupil tries to go down that escalator, at what speed (number of steps per second) should he walk down so that he does not remain in the same spot or go back up?

To solve such a problem it is necessary to use a number of strategies taken from the list presented earlier.

Here is the model:

Let x stand for the speed expressed as the number of steps per second. If $x > 0$, there is an upward movement, and if $x < 0$, there is a downward movement. Consequently, we have:

$$(2 + x)\ 90 < 0,$$

and the solution of this model will be:

$$180 + 90x < 0$$

$$90x < -180$$

$$x < -2.$$

To interpret the answer, it should be known that:

- the negative sign in -2 means that the pupil must go down;
- $x < -2$ means that the pupil will have to go down more than 2 steps per second.

The solution of the problem can be stated thus:

To avoid going back up or remaining in the same spot, the pupil must go down at a speed greater than 2 steps per second.

Normally, formulas are used to find an unknown. Since the other quantities are known, these formulas are considered equations or inequations of the first degree in one variable. The known values are considered the parameters in a specific situation and the unknown the variable. Supposing that the problem involves finding the height h of a rectangle with a known area A and base b, then:

- A would be considered a parameter,
- b would be considered a parameter,
- h would be considered a variable,

and consequently,

$$A = b \cdot h$$

will be considered an equation of the first degree in one variable.

The algebra program for the first cycle of secondary school covers much less than the term algebra implies. The program is intended to be a gradual introduction to the basics of algebra and a consolidation of previously acquired numerical concepts.

Many difficulties encountered in this discipline can be attributed to the high level of abstraction and generalization as well as to the precision which algebra requires. Nevertheless, the definitions, vocabulary, and techniques germane to the discipline are relatively simple, considering their affinities with the concepts used in the context of the operations on numbers.

In the first cycle, the emphasis is on solving simple equations and inequations of the first degree in one variable. However, handling equations and inequations also involves dealing with, among other things, sets expressed in expanded form, frames of reference, representation and properties of groups, rings or even bodies. Similarly, solving equations also involves the use of formulas in which the variables are no longer necessarily x, y, and z.

TERMINAL OBJECTIVE 3.1 **TO PERFORM basic operations on simple algebraic expressions.**

The aim of this objective is to enable the pupil to apply his basic knowledge of algebra so that he can master the tools which are needed for solving equations, inequations, or simple formulas.

The most complex propositional forms presented to the pupil will all be in one of the following forms:

$ax + b < c$

$ax + b = c$

$ax + b > c$

Consequently, it does not seem necessary to study very complicated like terms. The teacher should confine himself to simple like terms.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
3.1.1 TO IDENTIFY the terms, the coefficients, or the components of a given algebraic expression.	II	- This objective, as well as the others in this series, aims at teaching the pupil how to solve equations of the first degree in one variable. - Literal expressions are introduced at this stage. Example: $2a - b$; $2\pi r$; $\frac{(B + b)h}{2}$; πr^2. It is important to make the pupil understand that the introduction of "literal parts" will enable him to use the concepts he already knows in a great number of very different situations. - The introduction of literal expressions sometimes triggers a real mental block in a number of pupils... It is essential, therefore, to bear this in mind when these expressions are taught and to avoid using complex expressions. - The algebraic expressions presented to the pupil should be similar to: $7a - 3b + 2$ $4p - 2r$ (Terms in the form $\frac{a}{b}$ should not be used). - Definitions: - Term: the basic element of an algebraic expression which includes the product of constants and parameters (or variables). Examples: $2ab$, πr^2, $\frac{(B + b)h}{2}$ - Coefficient: generally, the numerical part which multiplies the "literal part of a term."

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		Examples: -2 in -2ab 4 in 4cd 2 in 2B - Exponent: the numerical (or literal) part which indicates how many times a parameter (or variable) is multiplied by itself. Example: 2 in πR^2
3.1.2 TO IDENTIFY similar (like) terms.	II	- Like terms are expressions that include the same parameters which carry respectively the same exponents.
3.1.3 TO ADD similar (like) terms.	II	- In the following algebraic expression: a + b + 3a - 2a + 3b, it is possible, using the commutative and associative properties, to group the like terms. The expression becomes a + 3a - 2a + b + 3b, then 2a + 4b, which is in fact the simplest equivalent expression. This is also known as "reduction of like terms." However, this terminology is ambiguous and should be avoided. - The addition of monomials is based essentially on the definition of an expression. Thus, 2a is a monomial defined by a + a. - The sum of 2a and 5a therefore is equal to $\underbrace{a + a}_{2a} + \underbrace{a + a + a + a + a}_{5a} = 7a$. - By applying the distributive property, the following result is obtained: 2a + 5a = (2 + 5)a = 7a. - Note: The expression 2a + 5a is no longer a monomial, and this cannot be considered an addition of monomials. It is rather a question of finding the term equivalent to 2a + 5a, which is a reduction. In this case, it is better to consider the operation as a search for the simplest equivalent algebraic expression, or, better still, as a calculation of the sum of the like terms.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
3.1.4 TO SUBTRACT similar (like) terms.	II	- Just like addition, the subtraction of monomials is a question of definition. - The subtraction of 7a from 5a is written 5a - 7a, which becomes an algebraic expression. Strictly speaking, it is a question of calculating the difference of like terms or else of finding the simplest equivalent algebraic expression.
3.1.5 TO MULTIPLY simple algebraic expressions.	II	- Simple monomials should be used. Example: $-2c$ and $4cd$, the product of which is $-8c^2d$. - As the preceding example makes clear, the pupil should be familiar with the following law of exponents: $\forall b \in \mathbb{Z}^*, \forall m, n \in \mathbb{Z}: b^m \cdot b^n = b^{m+n}$, or at least apply the definition of powers. Example: $a^2 \times a^3 = a^{2+3} = a^5$. $a^2 \times a^3 = (a \times a) \times (a \times a \times a) = a \times a \times a \times a \times a = a^5$
3.1.6 TO DIVIDE simple algebraic expressions.	II	- Simple monomials should be used. Example: The division of $-10b^2c$ by $2b$ is written $\frac{-10b^2c}{2b}$ or $-10b^2c \div (2b)$, the result of which is $-5bc$. - As the preceding example makes clear, the pupil should be familiar with the following law of exponents: $\forall b \in \mathbb{Z}^*, \forall m, n \in \mathbb{Z}: \frac{b^m}{b^n} = b^{m-n}$, or at least apply the definition of powers.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		However, factoring is a preferable method for solving this division. Examples: $\frac{-10b^2c}{2b} = \frac{-\cancel{2}^{1} \; 5 \; \cancel{b}^{1} \; b \; c}{\cancel{2}_{1} \; \cancel{b}_{1}} = -5bc.$
3.1.7 TO SIMPLIFY given algebraic expressions.	II	- Reducing an algebraic expression is a matter of adding the like terms it contains. This reduction can be done by using the associative and commutative properties. Here, it is possible to use the algorithms developed in Objectives 3.1.3 and 3.1.4 by applying them to algebraic expressions similar to those cited below: $3ab - 9c + 2ab + 7c - 12ab + f$ $2t - 3u + 7t + 5u - t.$ - Algebraic expressions in the form: $(3ab)(2ac)$, $\frac{2xy}{4x}$, etc. can also be reduced either by multiplication or by division.
3.1.8 TO FIND the numerical value of a given algebraic expression.	II	- This objective should be attained by using concrete situations and not through theoretical exercises. - Point out the effect of replacing the letters "a" or "b" with specific values in the expressions: 2a, 3a, and 5a.

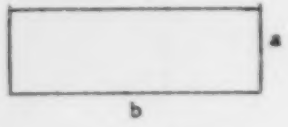
INTERMEDIATE OBJECTIVES	LEVELS	NOTES

a	2a	3a	5a
0	0	0	0
1	2	3	5
2	4	6	10
3	6	9	15
4	8	12	20
5	10	15	25
.	.	.	.
.	.	.	.
.	.	.	.

a + b, a + ab and a - 2b

a	b	a + b	a + ab	a - 2b
0	1	1	0	-2
1	0	1	1	1
3	2	5	9	-1
4	8	12	36	-12
5	10	15	55	-15
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

- This is a table which gives the numerical value of 2(a + b) and of ab for different values of a and b.



INTERMEDIATE OBJECTIVES	LEVELS	NOTES

Here, $2(a + b)$ represents the perimeter of the rectangle; ab represents the area of the same rectangle.

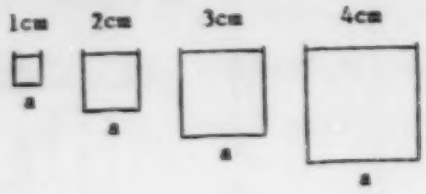
a	b	2(a + b)	ab
1	1	4	1
1	2	6	2
1	3	8	3
2	1	6	2
2	2	8	4
2	3	10	6
3	1	8	3
3	2	10	6
3	3	12	9
.	.	.	.
.	.	.	.
.	.	.	.

- It is worthwhile to consider what would come out of the expressions $2n$, n^2, and 2^n, for instance, if the value of n were to vary.

n	2n	n^2	2^n
0	0	0	1
1	2	1	2
2	4	4	4
3	6	9	8
4	8	16	16
5	10	25	32
.	.	.	.
.	.	.	.
.	.	.	.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES

- If the squares below are considered



then the expression 4a is used to find the perimeter of these squares. Indeed,

a	4a
1	4
2	8
3	12
4	16
.	.
.	.
.	.

ENRICHMENT: TO HANDLE more complex algebraic expressions, eventually using fractions or decimal numbers as numerical values.

TERMINAL OBJECTIVE 3.2 TO SOLVE problems involving common formulas or simple equations or inequations in one variable.

Constructing an equation or looking for the solution-set of that equation are steps which are often taken to solve a problem. The aim of the terminal objective covered here is to enable the pupil to develop a technique for solving equations. However, the operations should not be performed on equations or inequations taken out of their original context. Each equation or inequation which the pupil is asked to solve should come from situations drawn as often as possible from his familiar world. Here are two examples:

If a motorist does not drive over 100 km/h, what is the minimum amount of time it will take him to cover the 255 km distance between Québec and Montréal?

A clock is set at noon and 24 hours later it shows 12:48. What time did it show 3 hours after it was set?

Each situation should offer the teacher the opportunity to encourage the pupil to adopt the habit of using different problem-solving strategies. In Secondary III, the pupil will be able to refine his knowledge of the different techniques used to find the solution-set of an equation or an inequation. The technique stressed in Secondary II is one based on the concept of operators. This concept will be developed in the notes on the following intermediate objectives.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
3.2.1 TO FIND the solution set, given a reference set and an equation or inequation.	II	- These are a few definitions: - Proposition: A statement which is either true or false. - Reference set: The set which is being worked on and from which the elements come. - Variable: An undetermined term which may be replaced by any element of the reference set. - Propositional form: A statement which becomes a proposition if the variable is replaced by an element taken from the reference set. - Solution set: All the elements of the reference set which transform a propositional form into a true proposition. Examples: - Suppose: - the reference set: the pupils in my mathematics class. - the propositional form: "X measures over 1.75m." Then, P(X) will be a true proposition for... - Suppose: $E = \{0, 1, 2, 3, 4, 5\}$ $P(x): x + 1 \geq 3$ Then, P(x) will be a true proposition if x assumes the values 2, 3, 4, 5. - Attainment of this objective should enable the pupil to acquire the tools which he needs to "mathematize" a situation.
3.2.2 TO TRANSLATE a given work statement into a mathematical expression, and vice-versa.	II	

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Consider only simple cases. Here are a few examples: - The perimeter "P" of a rectangle is equal to twice the sum of the base "b" and the height "h." This translates into: $P = 2(b + h)$. - $A = \pi r^2$ This translates into: The area of a circle is equal to the product of the constant π and the square of the radius. - Peter has traveled twice the distance "d" between Québec and Montréal. This translates into: $2d$. - Three-quarters of the total amount of money "a" which Lise has. This translates into: $\frac{3}{4}$ a or $\frac{3a}{4}$. - Etc.
3.2.3 TO WRITE a simple equation or inequation in one variable from a given word sentence.	II	- This objective enables the pupil to make further progress in the process of mathematizing of a situation. - Only simple cases should be considered here. Here are some examples: - Peter is twelve years old. The sum of his and his friend's age is equal to 23. This translates into: $12 + x = 23$. - Julie has 34,25$. Once she pays back her debt she will have only 22,60$ left. This translates into: $34,25 - x = 22,60$. - Stephanie has a mass of 21 kg. The sum of her mass and Jean's mass is less than 40 kg. This translates into: $21 + x < 40$.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- In certain situations, constructing the equation or the inequation presents certain difficulties because it involves much more than a simple transposition of the written statement. Indeed, the inequation $250 < 100t$ and the equation $8(180 + x) = 24 \times 60 + 40$ are not necessarily obvious to those who read the two problems stated in the comments on Terminal Objective 3.2. Consequently, readers must proceed methodically to discover them. These are the steps which lead to the construction phase of an equation or an inequation: (a) Understanding the problem: - Read the problem several times and make certain every word is understood; - Reformulate the problem in your own words; - Identify the data; - Identify what is being sought; - Illustrate the data. (b) Developing a plan for solving the problem: - Eliminate non-pertinent data; - Find the missing data; - Construct or complete a figure; - Describe the principal conditions which the results should meet; - Break the problem down into smaller problems; - List the criteria that should guide your choices in a decision-making problem; - Look for one or several statements, one or several formulas, one or several properties,... - Look for a model.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		The different strategies which are part of the two stages described above can make it easier to reach the stage when the equation or inequation associated with the problem under consideration can be constructed.
3.2.4 TO IDENTIFY the reference set, given a problem in everyday life.	II	- The two objectives described above should have enabled the pupil to construct propositional forms usually called equations or inequations. - This objective requires that the pupil choose the appropriate reference set carefully. Several reference sets may be acceptable, but the pupil should be able to justify his choice. Examples: In the two problems stated in the notes on Objective 3.2, the reference sets could be respectively a period of time between 0 and 24 hours, or a specific hour between 0 and 24. In the escalator problem, the reference set could be a speed evaluated in steps per second, between -10 and +10, the minus and plus signs indicating the direction of the movement.
3.2.5 TO FIND the one value which satisfies an equation or a simple formula (given several values of a variable.)	II	- This objective is a special aspect of the more general case presented in Objective 3.2.1. Here, - the reference set provides "a number of values assigned to a variable;" - a propositional form is "a simple equation." - It is not a question of using the algorithms for solving an equation but rather of finding the values which satisfy the equation by using the substitution of values method.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES

Example: Suppose: $E = \{1, 2, 3, 4, 5, 6\}$ the reference set

$p(x)$: $2x = 6$;

then

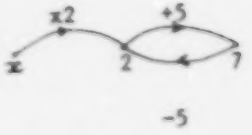
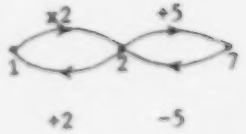
x	p(x)	
1	2 = 6	False
2	4 = 6	False
3	6 = 6	True
4	8 = 6	False
5	10 = 6	False
6	12 = 6	False

Therefore only the value 3 satisfies $p(x)$.

Example: If the area of a triangle can be covered with 520 small ceramic tiles 1 square centimetre in size, what is the height of that triangle if its base is 10 cm in length? Here the reference set could be the set of lengths between 0 and 520 centimetres.

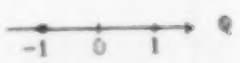
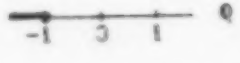
As the equation is $A = \frac{b \times h}{2}$, we will have, by estimating the degree of accuracy of the answer:

h	$A = \frac{b \times h}{2}$		
⋮			
100	$520 = \frac{10 \cdot 100}{2}$	False	
101	$520 = \frac{10 \cdot 101}{2}$	False	
102	$520 = \frac{10 \cdot 102}{2}$	False	
103	$520 = \frac{10 \cdot 103}{2}$	False	
104	$520 = \frac{10 \cdot 104}{2}$	True	Therefore, the value which satisfies $520 = \frac{10h}{2}$ is 104.
105	$520 = \frac{10 \cdot 105}{2}$	False	
⋮			

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
3.2.6 TO ISOLATE the variable in a simple equation or inequation.	II	- This is the time to explain to the pupil the different stages in the process of solving an equation or a simple inequation. - Use equations or inequations in the form $ax + b = c$, $ax + b < c$, $ax + b > c$, in which $a > 0$. - The algorithm which should be applied here is the one which involves the concept of operators. Some examples: (a) Isolate x in $2x + 5 = 7$. 1st step 2nd step Therefore $x = 1$. 3rd step  4th step  (b) Isolate x in $\frac{3x}{4} - 2 < 7$. 1st step 2nd step Therefore $x < 12$. 3rd step 4th step - The properties of operations can also be used to isolate the variable.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		Isolate x in $2x + 5 = 7$.
		$2x + 5 = 7$
		$(2x + 5) - 5 = 7 - 5$ (addition is well-defined)
		$2x + (5 - 5) = 7 - 5$ (associative property of addition)
		$2x + 0 = 2$ (closure property for addition)
		$2x = 2$ (identity element for addition)
		$\frac{1}{2}(2x) = \frac{1}{2} \times 2$ (multiplication is well-defined)
		$(\frac{1}{2} \cdot 2)x = \frac{1}{2} \times 2$ (associative property of multiplication)
		$1 \cdot x = 1$ (closure property of multiplication)
		$x = 1$ (identity element for multiplication)
		- ENRICHMENT: - Only the brighter pupils should be required to present justifications for the different steps followed in the preceding example.
		- By using the calculator and the algorithms for solving equations, it is easy to find the solution-set of an equation in the form: $\frac{3x}{4} - 0,79 = 5,43$. To use the calculator, it is necessary to write $0,75x - 0,79 = 5,43$. Hence $0,75x = 5,43 + 0,79$.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		$0,75x = 6,22$ $x = \frac{6,22}{0,75}$ $x = 8,2933333...$ Answer: $\{8,29\overline{3}\}$. Notes: - The objective is "to find the solution-set..." The teacher should therefore require that the pupil give his answer in set form. - If the calculator is used, the pupil should register all the significant figures which appear on the display screen, unless the question requires him to round off the answer. - A faster way of solving an equation in the form $ax + b = c$ with the pocket calculator would be to evaluate the equivalent equation: $x = \frac{c - b}{a}$. Find the solution-set of $2,3x + 4,79 = -13,12$. - By analogy with $ax + b = c$, it is known that: $a = 2,3$ $b = 4,79$ $c = -13,12$; - by substitution in $x = \frac{c - b}{a}$ we have: $x = \frac{-13,12 - 4,79}{2,3}$

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		hence, using the calculator: $x = -7,7869565...$ Answer: $\{-7,7869565...\}$.
3.2.7 TO REPRESENT a first degree equation or inequation graphically.	II	- In handling propositional forms (equations, inequations, or formulas) it is necessary to take the reference set into account because the value to be attributed to the variable will necessarily come from that set. - Graphing the propositional form makes it possible to see more clearly the relationship between the solution-set and the reference set. - Since the pupil is introduced to the set of real numbers only in Secondary III, it is suggested that the problems inherent to a representation in $\mathbb{Q}$ or in $\mathbb{Q} \times \mathbb{Q}$ not be raised. - In the second cycle of secondary school the teacher will explain the necessary distinctions. - Here are two examples: Example 1: $2x + 3 = 1$ $2x = -2$ $x = -1$  Example 2: $2x + 3 \leq 1$ $2x \leq -2$ $x \leq -1$  Locating the solution (Example 1) or the "critical value" (Example 2) can become difficult if that number is not a whole number. The pupil will be able to locate a rational number on a number line by using the technique presented in the note on Objective 2.1.4.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
3.2.8 TO ISOLATE the unknown in a common formula.	II	- In this objective, the algorithms used in Objectives 3.2.6 and 3.2.7 are applied to a formula. - Use simple common formulas. The formulas used in geometry are particularly suitable. - It is strongly advised that the context be specified. Example: If the perimeter of a triangle is determined by means of the formula $p = a + b + c$, find its side b. Example: If the area of a trapezoid is determined by means of the formula $A = \frac{(B + b)h}{2}$, find the longer base B.
3.2.9 TO MAKE a formula which represents a situation from: geometry, sports, science, the trades, etc.	II	- Here are some suggestions. (a) In geometry: - The sum of the measures of the interior angles of a triangle is 180°. $a + b + c = 180°$, a, b, and c being angular measures. - The ratio of the circumference of a circle to its diameter is equal to π. $\frac{c}{d} = \pi$, c being the measure of the circumference and d the measure of the diameter. - The perimeter of a square, a rhombus, a rectangle, a circle, etc. $p = 4c$

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		$p = 2b + 2h$ $p = \pi d$ etc. c being the measure of the side of a square, b being the measure of the base of a rectangle, h being the measure of the height of a rectangle, d being the measure of the diameter of a circle, p being the measure of the perimeter. - The area of a triangle, a square, a rhombus, a rectangle, a parallelogram, a trapezoid, a circle, etc. $A = \frac{b \times h}{2}$ $A = c^2$ $A = \frac{D \times d}{2}$ $A = b \times h$ $A = \frac{(B + b)h}{2}$ $A = \pi r^2$ b being the measure of the shorter base of a figure, B being the measure of the longer base of a figure, h being the measure of the height of a figure, c being the measure of the side of a square, r being the measure of the radius of a circle, A being the measure of the area.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The volume of a rectangular parallelepiped, etc. $V = l \times w \times h$ etc. l being the measure of the length, w being the measure of the width, h being the measure of the height, V being the measure of the volume. (b) In sports: The need to represent the links between certain variables by means of a formula is always present. Naturally, this need is also felt in sports. - Calculating a baseball player's pitching average: $\frac{\text{number of runs scored}}{\text{number of innings pitched}} = \text{average},$ hence $\frac{R}{i} = a.$ - Calculating a baseball player's batting average: $\frac{\text{number of hits}}{\text{number of official at bats}} = \text{average},$ hence $\frac{h}{b} = a.$ - Calculating a hockey goalkeeper's average: $\frac{\text{number of goals against}}{\text{number of games played}} = \text{average},$ hence $\frac{g}{p} = a.$

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Calculating a golf handicap. $h = 0{,}85\ (a - 72)$, h being the player's handicap, and a being the player's average. (c) In general science: Several phenomena of general interest can be presented under this heading. Here are some examples: - Relationship between blood pressure and age: $p = 100 + \frac{a}{2}$, a being the age expressed in number of years, and p being the pressure. - Link between the mass and the height of an adult: $M = 98{,}218805\ (h - 1{,}016)$, h being the height expressed in metres, and M being the mass expressed in kilograms. - Relationship between the distance traveled by a free-falling body and the time it takes to travel that distance: $d = 5t^2$ d being the distance, and t being the time. - Conversion of Farenheit degrees into Celsius degrees: $C = \frac{5}{9}\ (F-32)$, F being the temperature in Farenheit degrees, and C being the temperature in Celsius degrees.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Distance in relation to rate and time: $d = r \times t$, d being the distance, r being the rate, and t being the time. (c) In business: - Calculating annual interest: $I = pr$, I being the interest, p being the principal, and r being the rate. - Calculating the interest for a given number of months: $I = pr \cdot \frac{x}{12}$ $I = \frac{24f}{p(n-1)}$, I being the simple interest rate, p being the outstanding unpaid principal, r being the interest rate, x being the number of payments, f being the finance fees, and n being the number of monthly payments.

GENERAL OBJECTIVE

4

TO PROMOTE participation in activities involving the concept of measure or of location.

THEME: Measure and location

TIME: 8% in Secondary I
8% in Secondary II

In elementary school the pupil acquired a certain knowledge of the International System of Units. The aim of this section is to enable him to further develop his knowledge of measure by giving him the opportunity to apply that knowledge to concrete situations.

Simply by measuring the familiar objects found in his environment, the pupil will be able not only to develop his measuring skills but also to have a better perception of the world which surrounds him.

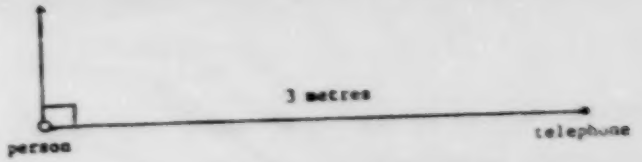
While working with measures, the pupil should learn to select the measuring unit which is appropriate to a given situation. The distance between two towns should be measured in kilometres and not in metres; a person's height should be expressed in metre(s) and not in centimetres, and so on.

Using the SI units in measurement activities should become second nature for the pupil, and to that end he should develop a reference system based on repeated exercises. It is not recommended that Imperial to SI conversion tables be made available to the pupil, and all references to the imperial system should be avoided. The ultimate aim of this objective is to teach the pupil to think metric.

The teacher should insist that the pupil follow faithfully the rules respecting the writing of units, symbols, etc.

Location constitutes an important aspect of human activity. It involves finding the place of an object in a network, the position of a person in a room, the location of a building in a city, the position of a town on a map, the position of a ship on a river or on the ocean, the position of an airplane in the air, and so on.

To determine location it is necessary to use a reference system. The first one that comes to mind is the Cartesian system, which is used especially in analytical geometry and cartography. However, this system is neither the only one in existence nor the most natural. If someone says that there is a telephone located three metres to his right, it would be possible to draw the following representation:



At least two information items can be deduced from this statement:

- the distance separating the person from the telephone is 3 metres;

- the telephone is located to the right of that person.

It becomes possible to symbolize this information by the pair (3, right). By attributing a value of 0^0 to the position facing the person and by measuring the angles clockwise, it becomes possible then to represent the position of the telephone by the pair (3, 90). It can be said that the reference system used here is a system of polar coordinates in which the speaker considers himself as the pole (center) from which the reference has been established.

It is important for the teacher to realize that the system of polar coordinates is the most natural. It is said, for instance, that Québec is located 256 km north east of Montréal. On the basis of this observation it seems useful to mathematize the human activity of location so as to interpret it better. In this last example, the mathematization of the situation would be expressed by the pair (256,45).

The aim of this objective is to develop the pupil's sense of observation and orientation by using the tool made available to him by mathematics, namely, representation by means of systems of coordinates.

TERMINAL OBJECTIVE 4.1 TO APPLY the concept of angular measure.

It would be sensible to define what an angle is before thinking of measuring it. There are many different opinions as to the nature of an angle and the definitions which are given very often vary according to the particluar aspect which is emphasized. These are some definitions of an angle:

- "It is the union of two rays which have the same endpoint."
- "It is the displacement of a ray by rotation."
- "It is the intersection of two half planes."
- "It is the opening between two rays emanating from the same point."

All these definitions are good, but only the first two will be adopted. The first definition will be adopted because it presents the angle as a subset of a plane and because it makes it possible to consider the three regions demarcated by the angle: the angle itself, its inside, and its outside. The second definition will be adopted because it presents a dynamic aspect of the angle, an aspect which is particularly stressed in transformational geometry and in the use of polar coordinates to locate objects on a radar screen.

The various measurement activities proposed to the pupil will enable him to assimilate the concept of angle. These activities are of particular importance here and the teacher should start with concrete situations in order to attain the intermediate objectives.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
4.1.1 TO ESTIMATE in degree measure of an angle to a given accuracy.	I	- Define "degree" as the measure of an angle at the center which intercepts on the circumference an arc whose length is equal to $\frac{1}{360}$ of that circumference. - The pupil can be required to aim at a ±10 degree of accuracy. - Estimating the measure of an angle is often a difficult and tricky activity. Example: Estimate in degrees the measure of ∠ to a ±10° degree of accuracy (to the nearest 10 degrees). - The teacher should not limit himself to estimations such as the one in the preceding example. He should as often as possible use objects taken from the pupil's environment.
4.1.2 TO MEASURE a given angle to the nearest degree, using a protractor.	I	- Toward the end of elementary school, the pupil learns to use a protractor to "measure an angle in degrees." - Insist that the pupil be able to measure a given angle with the greatest precision possible (to the nearest degree). - It could be very practical to use a level square to measure with precision the value of an angle formed by two intersecting planes. - ENRICHMENT: - TO DEFINE the minute and the second as units of angular measure.
4.1.3 TO DRAW an angle of a given degree measure, using a protractor.	I	- Insist that the pupil learn to draw a given angle with the greatest precision possible (to the nearest degree).

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
4.1.4 TO DRAW to the nearest 5° angles of 30°, 45°, 60°, 90°, and 120°, without a protractor.	II	- The aim of this objective is to develop the pupil's ability to estimate the measure of an angle. In order to reach this objective, the approximation should not exceed a margin of error of 5°.

TERMINAL OBJECTIVE 4.2 TO APPLY the concept of measure in situations involving the measurement of segments.

In order to condition the pupil to think in metric, it is recommended that the teacher start with concrete situations by having him measure familiar objects: personal height and weight, a table, a door, a window, a house, a car, the distance between two objects, etc.

It is advisable to avoid all references to the Imperial system and to forbid the pupil to use conversion tables. The use of such tables is liable to keep the pupil from developing a metric reference system.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
4.2.1 TO NAME the preferred unit of length in various situations related to everyday life.	I	- In elementary school, the pupil should be able "to use the SI symbols correctly." The symbols dealt with in this objective are: mm, cm, m, and km. - The basic unit for measuring lengths is the metre whose symbols is m. - Use familiar objects as much as possible. - These are some examples: - The millimetre should be used for measuring the dimensions of nails, screws, postage stamps, films, lenses, drill bits, wood chisels, sheets of paper, etc. - The centimetre should be used for measuring body dimensions, the width of a piece of material, dress patterns, lengths of objects such as laces, scarves, etc. - The metre should be used for measuring the dimensions of houses and buildings, the length of a piece of material, the height of waterfalls and mountains, the dimensions of sports fields, etc. - The kilometre should be used for indicating distances on road signs, the length of streams, the dimensions of lakes, of continents, etc.
4.2.2 TO ESTIMATE in centimetres or in metres, to a given degree of accuracy, a dimension of an object.	I	- This objective will enable the pupil to become familiar with the degree of accuracy of the SI units of measure by using those units in his environment.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Select real objects rather than their representation.
		- ENRICHMENT: - TO USE the units: mm, cm, dm, m, dam, hm, and km.
4.2.3 TO MEASURE in millimetres, centimetres, or metres, to a given degree of accuracy, a dimension of an object.	I	- The pupil should take those measurements with a metric ruler. - Use familiar objects as much as possible.
4.2.4 TO ESTABLISH the relationships among the common units of length.	I	- Present the prefixes and the rules respecting their usage. - The pupil should develop the ability to use the different units cited in the objective interchangeably and with equal ease. - ENRICHMENT: - TO PERFORM these transformations on the following units: mm, cm, dm, m, dam, hm, km.

TERMINAL OBJECTIVE 4.3 TO APPLY the concept of measure in situations involving the measurement of area.

In order to condition the pupil to think in metric, it is recommended that the teacher start with concrete situations by having him measure familiar objects: the area of a floor, the area of a field, the surface area of a country, etc.

It is preferable to avoid all references to the Imperial system and to forbid the pupil to use conversion tables. This policy is intended to help the pupil learn to think in metric.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
4.3.1 TO DETERMINE the area of a plane geometric figure using a non-standard unit of area.	II	- It would be desirable to draw the distinction between the concepts of area and surface: . A surface is the region of a plane demarcated by line segments or arcs. . An area is the measure of the surface by means of a unit of measure. - The following example should make that distinction clearer: - "The length of a segment" This is a segment ————, and its length is 3 cm. - "The area of a surface" This is a surface , and its area is $3cm^2$.
4.3.2 TO NAME the preferred unit of area in a given situation.	II	- The "non-standard" unit of area mentioned in the objective could be called: square unit, rectangular unit, triangular unit, etc. - In elementary school, the pupil should "use the SI symbols correctly." The symbols considered in this objective are: cm^2, dm^2, and m^2. - This objective should enable the pupil to become familiar with the degree of accuracy of the SI units of measure by using them in his environment. - The following units should be used: mm^2, cm^2, m^2, hm^2, and km^2. - The unit ha (hectare) is to be preferred when measures are taken in hm^2 (square hectometre).

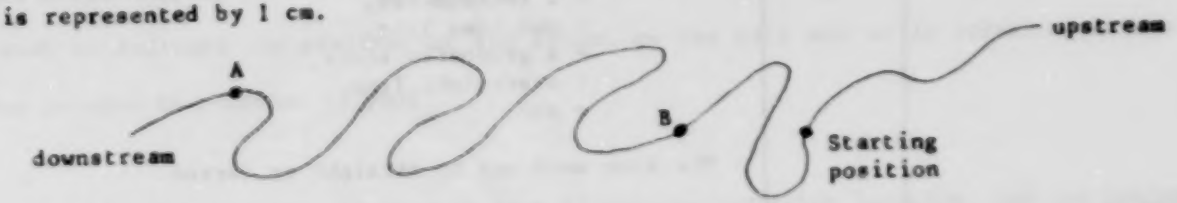
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- These are some examples: - The <u>square centimetre</u> unit should be used for measuring the surface of playing cards, of the sides of small boxes, of geometric figures constructed on paper, etc. - The <u>square metre</u> unit should be used for measuring the surface of rooms, walls, draperies, patios, sections of road to be paved, etc. - The <u>hectare</u> unit should be used for measuring the surface of parks, municipalities, etc. - The <u>square kilometre</u> unit should be used for measuring the surface of deserts, large forests, lakes, provinces, countries, etc.
4.3.3 TO DETERMINE how many times a given unit of area is smaller or larger than another unit of area.	II	- The area of the square can be used to deduce the techniques for converting the units of area. Example: $100\ cm^2 \times 100 = 10\ 000\ cm^2$ $1\ m^2 \times 1 = 1\ m^2$ hence $1\ m^2 = 10\ 000\ cm^2$. - Use the following units: cm^2, m^2, and km^2. - ENRICHMENT: - TO USE the following units: mm^2, cm^2, dm^2, m^2, dam^2, hm^2, and km^2.

TERMINAL OBJECTIVE 4.4 TO LOCATE an object on a line.

Attainment of this objective is useful in the context of a certain mathematization of a situation and of the interpretation which can be given to the information drawn from that situation. Some examples:

Locate a boat that has been drifting for 5 hours on a stream flowing at a speed of 2.7 km per hour.

Here 1 km is represented by 1 cm.



What would be the position of the boat after five hours in relation to the castle located at point A and the tower at point B?

Such location activity can involve integers by representing an upstream - downstream movement by a positive number, and a downstream - upstream movement by a negative number.

Road maps or sea charts can be used as support for locating an object on a line. It is also strongly recommended that the teacher use concrete situations which can be understood by the pupil.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
4.4.1 TO DETERMINE the position of an object on a line, given the pertinent information.	I	- Determining location is an important everyday activity. This objective aims to develop the pupil's ability to locate an object, a place, an event on: - a street, - a river, - a thermometer, - the time line, - a graduated scale, - a straight line, - etc. - The line used may be straight or curved.
4.4.2 TO DETERMINE the position of a point on an axis, given the pertinent information.	I	- This objective is similar to the preceding one, and it makes it possible to mathematize the events presented during the learning activities used to attain Objective 4.4.1. The use of the point and of an axis makes it possible to introduce the pupil to mathematization from a graph.

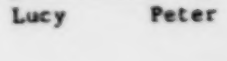
TERMINAL OBJECTIVE 4.5 **TO LOCATE an object on a surface.**

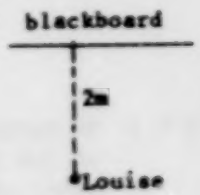
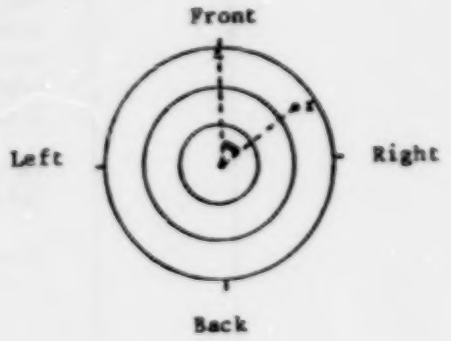
Attainment of this objective is useful in the context of a certain mathematization of a situation and of the interpretation which can be given to the information drawn from that situation.

The use of a Cartesian or polar system depends on the situation considered and on the kind of information desired. In a given situation, do you wish to locate Sept-Iles on a map or to indicate its position in relation to Rimouski? Do you wish to indicate the position of the school on the city map or in relation to your home? Where is your nearest neighbor located in relation to you?

A great number of activities can be derived from situations involving location, and the teacher should encourage these activities to help the pupil develop a better spatial perception. Consequently, the pupil should be given the opportunity to express the location of an object symbolically and to become accustomed to decoding symbolic information to find the position of an object on a plan or on a map.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
4.5.1 TO DETERMINE the position of an object on a surface, given the pertinent information.	II	- In elementary school, the pupil is taught "to develop and to apply techniques for solving problems related to spatial relationships." This activity relies very much on intuition. - Determining location is an important everyday activity. This objective aims to develop the pupil's ability to locate an object, a place, an event on: - a map, - a grid, - a network, - a screen, - a plan, - etc.
4.5.2 TO DETERMINE the position of a point on a plane, given a set of axes (Cartesian, or not) and some pertinent information.	II	- This objective is similar to the preceding one, and it makes it possible to mathematize the events presented during the learning activities. The use of the point and the plane makes it possible to introduce the pupil to mathematization from a plane. - It is important that the pupil be able to use the following vocabulary: - Cartesian plane, - Non-Cartesian plane, - quadrant, - the x-axis, - the y-axis, - origin, - pair

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- abscissa, - ordinate, - coordinates.
4.5.3 TO DETERMINE the coordinates of the image of a point under certain reflections or rotations in the Cartesian plane.	II	- The image of a point (a, b) in the Cartesian plane can be found by considering: - the x-axis as axis of symmetry, - the y-axis as axis of symmetry, - the origin as center of symmetry. - ENRICHMENT - TO USE the "y = x" axis (the bisector of the first quadrant) as axis of symmetry.
4.5.4 TO DETERMINE the position of an object in a plane, given a set of polar coordinates.	II	- In several situations, the system of Cartesian coordinates is not enough for effectively locating an object. In such cases, the system of polar coordinates should be used to supplement the other system. Radar and sonar are examples of measuring instruments which make use of of polar coordinates. - The use of a map and a compass in a given situation can be an opportunity to introduce the system of polar coordinates. A map, a protractor, and a pair of compasses can also be used for that activity. (The compasses make it possible to transpose the basic unit on the map in order to evaluate a given distance.) - In many familiar situations, the pupil orients himself by means of polar coordinates: Examples: - Lucy is standing at distance of one metre to the left of Peter. This situation can be illustrated thus:  and symbolized by the pair (1, left).

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The blackboard is 2 metres behind Louise. This situation can be illustrated thus: The pair (2, behind) can be used effectively and unequivocally to locate the blackboard in relation to Louise. 
		- To locate an object X by means of polar coordinates, three conditions are necessary: 1. To know the position of the origin (the pole) in the plane; 2. To measure the distance separating the object from the pole; this measure constitutes the abscissa of the pair; 3. To evaluate the angle of rotation which makes it possible to reach the object; this angular measure constitutes the ordinate of the pair. 

GENERAL OBJECTIVE

5

TO DEVELOP in the pupil the habit of using geometric concepts, relations, and properties.

THEME: Geometry

TIME: 20% in Secondary I
17% in Secondary II

Geometry should be taught in a concrete and dynamic way in the first cycle of secondary school. For this reason, this program begins with two intermediate objectives on transformational geometry. After a first glance at the world of translations, rotations, reflections, and homothetic transformations (enlargements or reductions), the pupil is encouraged to discover for himself the properties of these plane transformations and the invariant properties which they preserve.

The knowledge the pupil acquires of plane transformations will enable him to construct a variety of geometric figures (Terminal Objective 5.3), to identify the different properties of triangles and quadrilaterals (Terminal Objective 5.4), and to resolve problems in everyday life in which units of measure of length and of surface area (Terminal Objective 5.5) are involved.

Since the representation of things plays an important part in human activity, it is necessary for the pupil to look first at concrete situations in the light of plane transformations in order for him to find out the properties of some of the figures involved and the invariant properties preserved in these situations.

It is essential that the teacher make every possible effort to integrate the different aspects of geometry in his teaching by using a methodology which elicits the creativity, dynamism, and active participation of the pupil.

The notes on each intermediate objective are intended to help the teacher understand clearly the approach he should adopt in teaching this course.

TERMINAL OBJECTIVE 5.1 TO REPRODUCE a plane figure using geometric transformations.

The purpose of this objective is not to engage the pupil in a formal study which has little to do with reality, but to help him recognize the numerous advantages of exploring the two-dimensional world using geometric transformations.

The pupil should be able to reproduce a plane figure by using one or more geometric transformations (translations, rotations, reflections, size transformations, etc.), and by tracing, sliding, turning, or flipping an object using a pantograph or some other technique of geometric construction.

The symbols used should be limited in number to those absolutely necessary for the pupil to develop a clear understanding of the situation being studied.

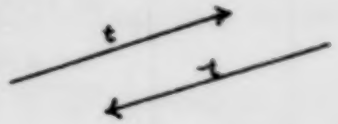
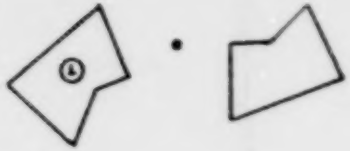
The geometric transformations presented in this objective will, among other things, enable the teacher to define the following concepts:

- parallelism (through translation),
- perpendicularity (through reflection),
- angle (through rotation),
- congruence (through isometric transformations),
- similarity (through homothetic transformations: enlargements or reductions)

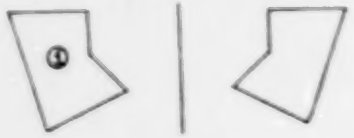
These transformations can also be regarded as integrating elements in geometry. The study of translations, rotations, and reflections will provide the pupil with a valuable tool which will make it easier for him to identify the different properties of geometric figures and to understand the formulas for perimeter and area. It is thus possible for the teacher to integrate transformational geometry and traditional geometry through dynamic teaching methods.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.1.1 TO DRAW the image of a figure obtained by a translation, rotation or reflection.	I	- Translations, rotations, and reflections can be found everywhere in the pupil's environment: a box moving along a conveyor belt, an airplane propeller, a mirror, etc. It is important that the teacher first introduce situations that are familiar to the pupil and then proceed gradually to the underlying mathematical concept. - <u>Translations</u>: - A translation could be defined as the moving of all the points in a figure in the same direction in a plane. - The image of figure 1 below is obtained by a translation: - The image of figure 1 below is not obtained by a translation:

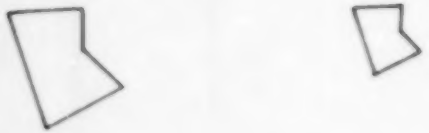
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The pupil should be encouraged to find examples of translations in his own experience. - The pupil may do the recommended activities using templates which are provided or tracing paper. However, he should be discouraged from using tracing paper once he has sufficiently mastered the concept of translation. - The pupil should be able to use the following words and expressions correctly: - translation, - translation arrow, - translation movement, - original figure, - image. - The pupil should also be able to: - recognize if two arrows represent the same translation, - recognize the invariant properties, - represent a translation using an arrow, or a point and its image.

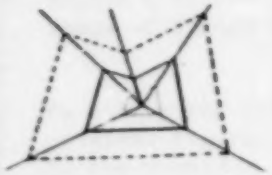
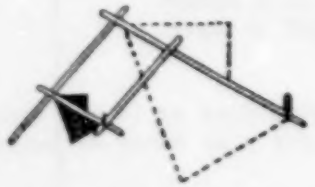
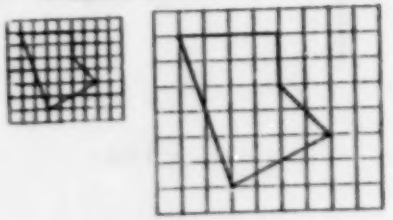
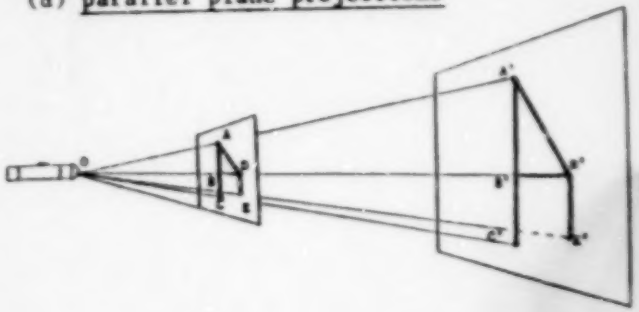
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- After a number of exercises, the teacher could introduce a limited number of symbols by designating the translations "t," "-t," "u," "-u," etc., while continuing to use the translation arrow or a point and its image to define the translation. - The translation t and its inverse translation -t are shown below:  - The formal terminology used should not be more difficult than this in the first cycle of secondary school. - <u>Rotations</u>: - A rotation could be defined as a turning, in a plane, about a point called the centre of rotation. - The image of figure 1 below is obtained by a rotation: 

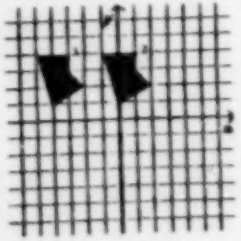
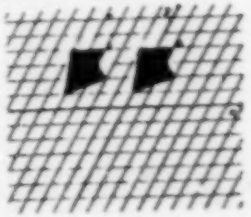
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The transformation below is not obtained by a rotation: - The pupil should be encouraged to find examples of rotations in his own experience. - The pupil may do the recommended activities using templates which are provided or tracing paper. However, he should be discouraged from using tracing paper once he has sufficiently mastered the concept of rotation. - The pupil should be able to use the following words and expressions correctly: - rotation, - rotation arrow, - rotation movement, - centre of rotation, - original figure (pre-image), - image.

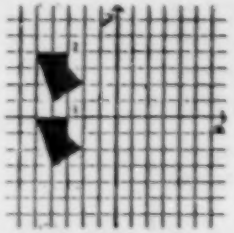
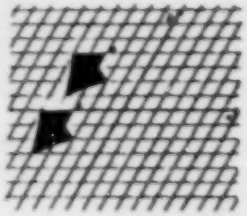
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The pupil should also be able to: - recognize if two arrows represent the same rotation, - recognize the invariant properties, - represent a rotation using an arrow and a centre of rotation. - After a number of exercises the teacher could introduce a limited number of symbols by designating the rotation "r" and its inverse rotation "-r," while continuing to use the centre of rotation and rotation arrow to define the rotation. - The arrows below represent the rotation "r" and its inverse rotation "-r." - The formal terminology used should not be more difficult than this in the first cycle of secondary school. - Reflections: - A reflection could be defined as a flip about an axis. - The image of figure 1 below is obtained by a reflection: 

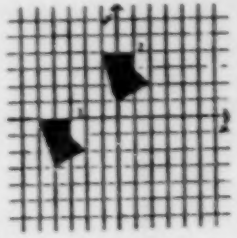
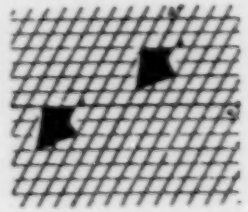
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The transformation below is not obtained by a reflection: - The pupil should be encouraged to find examples of reflections in his own experience. - The pupil may do the recommended activities using templates which are provided or tracing paper. However, he should be discouraged from using tracing paper once he has sufficiently mastered the concept of reflection. - The pupil should be able to recognize and use the following words and expressions correctly: - reflection, - symmetry, - axis of symmetry, - original figure (pre-image), - image. - The pupil should also be able to: - recognize if a straight line is an axis of reflection, - locate the line of reflection, given the original figure and its image, - recognize the invariant properties, - find the axis(-es) of symmetry of a figure.

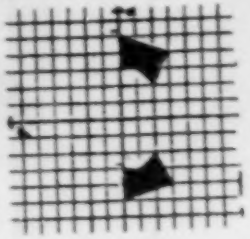
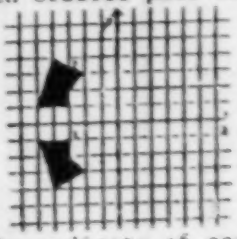
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.1.2 TO DRAW the enlarged or reduced image of a figure.	I	- Enlargement or reduction can be defined as the expansion or contraction of a figure in a plane in such a way that the measures of corresponding angles remain equal and the ratios of the lengths of corresponding sides remain equal. - An enlargement is shown below: - A reduction is shown below:  - The pupil should be encouraged to find examples of enlargements and reductions in his own experience. - The pupil should be able to use the following words and expressions correctly: - enlargement, - reduction, - original figure (pre-image), - image.

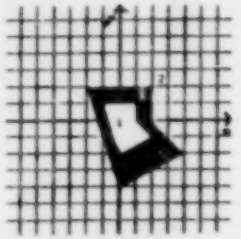
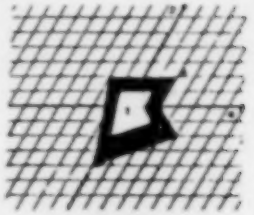
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The pupil should also be able to: - recognize the invariant properties, - recognize the scale factor (ratio). - The teacher should know that such enlargements and reductions are simply size transformations. - It would be preferable to give the pupil exercises involving only simple scale factors. - "... two times as big ..." - "... three times as big ..." - etc. - The teacher can present different techniques to the pupil: (a) the spider's web  (b) the pantograph  (c) square grids  (d) parallel plane projections 

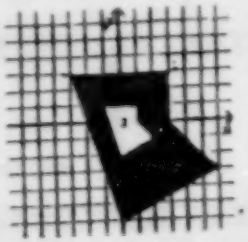
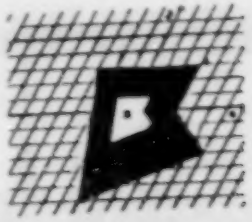
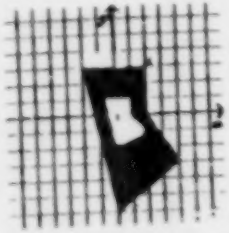
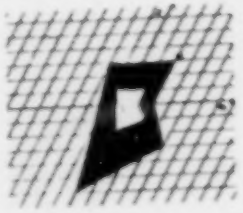
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.1.3 TO DRAW the image of a figure by performing a given operation on its coordinates.	II	- A system of axes, Cartesian or otherwise, is needed to locate the image of a given figure. Certain concepts concerning location which were presented in General Objective 4 should be reviewed here. - It would also be useful to review the geometric transformations studied in Secondary I. This is best done by means of the activities suggested in the present objective. - This objective should enable the pupil to find the image of a figure obtained by translation, reflection, enlargement, or reduction. Examples: (a) <u>by translation</u>: - Draw the image of Figure 1 by adding the ordered pair (4,0) to each ordered pair in the original figure.  Figure 2 is the image of Figure 1.  Figure 2 is the image of Figure 1.

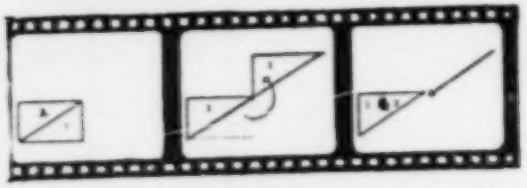
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		By adding the ordered pair (a, 0) to each ordered pair in the original figure, the figure is moved in a direction parallel to the x-axis. - Draw the image of Figure 1 by adding the ordered pair (0, 4) to each ordered pair in the original figure.  Figure 2 is the image of Figure 1.  Figure 2 is the image of Figure 1. By adding the ordered pair (0, b) to each ordered pair in the original figure, the figure is moved in a direction parallel to the y-axis.

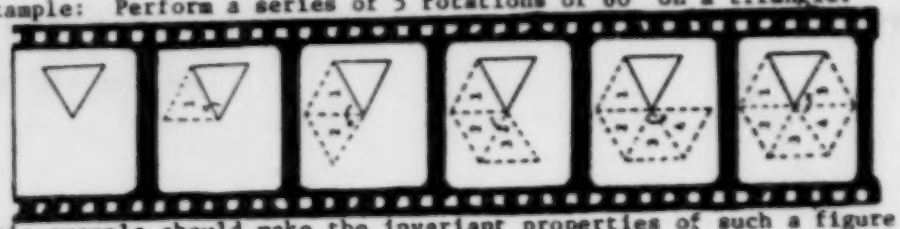
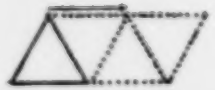
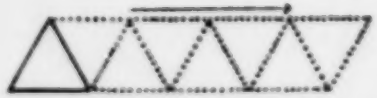
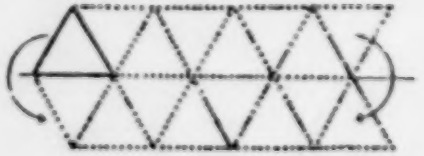
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Draw the image of Figure 1 by adding the ordered pair (4, 4) to each ordered pair in the original figure.  Figure 2 is the image of Figure 1.  Figure 2 is the image of Figure 1. By adding the ordered pair (a, a) to each ordered pair in the original figure, the figure is moved in a direction parallel to the axis described by $y = x$. - It should also be pointed out to the pupil that the sum of the ordered pair (a, b) and each ordered pair designating a point in the original figure will produce a translation in the plane.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		(b) <u>by reflection</u>: - Draw the image of Figure 1 obtained by replacing the abscissa of each ordered pair with its negative.  Figure 2 is the image of Figure 1. By replacing the abscissa of each ordered pair with its negative, the x-axis becomes an axis of symmetry. - Draw the image of Figure 1 obtained by replacing the ordinate of each ordered pair with its negative.  Figure 2 is the image of Figure 1. By replacing the ordinate of each ordered pair with its negative, the y-axis becomes an axis of symmetry.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Another possible activity would be to have the pupil draw the image of a given figure obtained by replacing both coordinates of each ordered pair with their negatives, and to have him identify the new axis of symmetry. - The teacher could also help the pupil find the image of a given figure, first in relation to the x-axis and then in relation to the y-axis. He could then help the pupil observe that the process involved is a rotation of 180° about a centre which is located at the origin of the system of axes. (c) <u>by enlargement or reduction</u>: - Draw the image of Figure 1 obtained by doubling its coordinates.  Figure 2 is the image of Figure 1.  Figure 2 is the image of Figure 1.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Draw the image of Figure 1 obtained by dividing its coordinates by 3.  Figure 2 is the image of Figure 1.  Figure 2 is the image of Figure 1. It should be brought to the pupil's attention that the multiplication of the coordinates of each point in a figure produces an enlargement of the figure, whereas the division of the coordinates produces a reduction. - Draw the image of Figure 1 obtained by multiplying the x-coordinates (abscissae) of each ordered pair by 2 and the y-coordinates (ordinates) by 3.  Figure 2 is the image of Figure 1.  Figure 2 is the image of Figure 1.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		It should be brought to the pupil's attention that the multiplication by different numbers of each member of the ordered pairs designating the original figure produces either an enlargement or a reduction which is a distortion of the original. It is thus not possible to establish a constant proportion between the lengths of corresponding segments in the original and final figures.
5.1.4 TO DRAW the image of a figure obtained by a composite of translations, rotations, or reflections.	II	- This objective is an extension of Objective 5.1.1. - The following situations should be explored: - an object and a mirror; - an object and two parallel mirrors; - an object and two mirrors forming a 90°, 45°, 30° angle, etc. - Formal terminology or symbols which might interfere with the clear understanding of the situations being studied should not be used. - Geometric transformations should be described using graphic representations and concise statements. Example: Perform a rotation and a translation on Triangle 1 in such a way that it is superimposed onto Triangle 2.  It is then easy to see that if the area of the rectangle is given by the formula A = b x h, the area of the triangle is A = $\frac{b \times h}{2}$.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- An interesting activity would be to construct different figures and to find their properties. Example: Perform a series of 5 rotations of 60° on a triangle.  This example should make the invariant properties of such a figure very clear. - A series of geometric transformations makes it possible to tile a portion of a plane. - original figure - rotation - translation  - translation  - reflection 

TERMINAL OBJECTIVE 5.2 TO IDENTIFY the image of a relation, given this relation.

After having constructed the image of a given figure which has undergone a geometric transformation, the pupil should be able to distinguish the image of this figure from the images of other figures. The use of geometric transformations should enable him to recognize the properties of a variety of geometric figures and to discover the different formulas for calculating the areas of certain polygons.

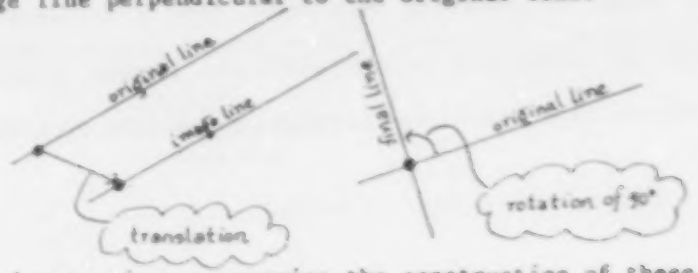
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.2.1 TO IDENTIFY in a given set of figures that (those) which is/are congruent to the given one.	I	- The aims of this objective are: - to develop keener powers of observation in the pupil, - to enable him to recognize congruences. - In elementary school, the pupil has learned "to perform geometric transformations which maintain the dimensions of the figures unchanged, and to describe these transformations." - The pupil should be made aware of the invariant properties, which may be: - the preservation of angle measures; - the preservation of shapes; - the preservation of parallelism; - the preservation of perimeters; - the preservation of areas; - etc. - The pupil should be made aware that the expression "congruent" signifies having: - the same shape, - the same lengths for corresponding sides, - the same measures for corresponding angles. - Geometric transformations involve the pupil actively in the study of geometry and enable him to develop his psychomotor skills.

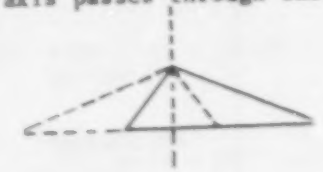
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Very few symbols are necessary for the pupil to achieve this objective: - a translation can be represented by a "straight arrow;" - a rotation can be represented by a "curved arrow" and a point designating the centre of rotation. - ENRICHMENT: A certain amount of formal terminology could be introduced to enable the pupil to better describe the rotations and translations involved.
5.2.2 TO IDENTIFY in a given set of figures that (those) which is/are similar to the given one.	II	- The aims of this objective are to: - develop the pupil's powers of observation, - develop the pupil's intuitive understanding of the concept of similarity which will be studied in the second cycle. - The pupil should be made aware of the invariant properties, which may be: - the preservation of angle measures, - the preservation of parallelism. - The pupil should be made aware that the terms enlargement or reduction signify the presence of all of the following conditions: - the same shape, - proportional lengths for corresponding sides, - the same angle measures for corresponding angles.

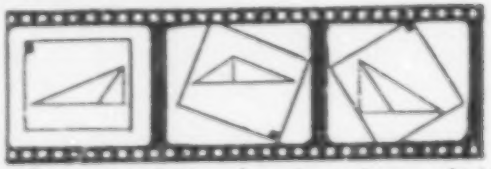
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Numerous examples of similarity exist: - a photographic enlargement; - a map of the stars and their images as seen through a telescope; - a cell and its magnified image as seen through a microscope; - an object and its image seen with the aid of a heliograph (heliostat); - a car and a small-scale model of it; - a model and the real object it represents; - pictographs using proportional motifs; - etc.

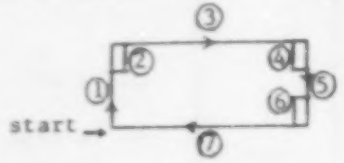
TERMINAL OBJECTIVE 5.3 TO CONSTRUCT various plane geometric figures.

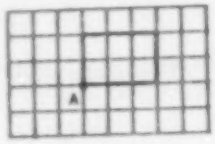
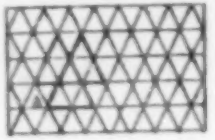
The geometric transformations presented in Terminal Objectives 5.1 and 5.2 are of use to the pupil in that they enable him to construct a variety of plane geometric figures. By presenting the construction of figures in terms of transformations, the teacher is able to integrate the different types of geometry in his teaching, and at the same time, to avoid the danger of covering this topic from a strictly technical point-of-view, for the use of geometric transformations makes the properties of the different figures constructed almost immediately apparent. Some teachers might therefore prefer to teach this objective in conjunction with the following one, Objective 5.4.

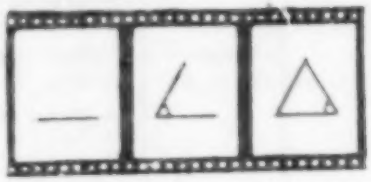
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.3.1 TO CONSTRUCT a line parallel or perpendicular to a given line, given certain conditions.	I	- The teacher should make sure that the pupil is aware that it is possible to obtain a straight line parallel to a given line by a simple translation of the latter. Also, a rotation of 90° will produce an image line perpendicular to the original line.  - The instructions concerning the construction of these parallel lines could vary as follows: - draw, freehand, - use a ruler only, - use a set square and a ruler, - use a protractor and a ruler, - use a set square, compass, and ruler, - etc. - The degree of accuracy of the pupil's construction will depend to a large extent on the instruments he is instructed to use. This factor should be taken into account when his work is being evaluated.

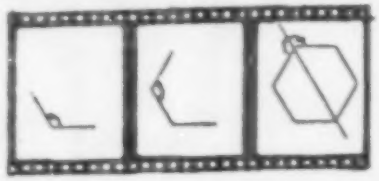
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- ENRICHMENT: Use a compass to construct a perpendicular to a straight line. This perpendicular must pass through: - a point on the line, - a point outside the line.
5.3.2 TO CONSTRUCT a triangle given the measure of the three sides, the measure of two angles and the included side, or the measure of two sides and the included angle.	I	- The pupil should do this construction using the following instruments: - a ruler, - a protractor, - a compass. - The pupil should be encouraged to construct his figures as accurately as possible.
5.3.3 TO CONSTRUCT an altitude of a triangle.	I	- The altitude of a triangle can be determined by using a reflection in which the axis passes through one of the vertices of the triangle.  - The axis needed is the one which makes it possible to construct the base of the image triangle in the same straight line as the base of the original triangle. A mirror can be used to determine this axis (altitude).

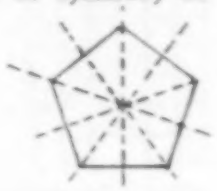
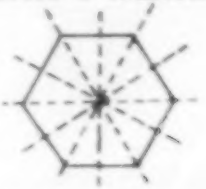
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- This altitude can also be constructed: - without using any instruments* [freehand], - using a ruler and a protractor, - using a ruler and a compass, - etc. - The pupil should be encouraged to construct his figures as accurately as possible. - The pupil can be asked to construct the three altitudes of a triangle in a relatively simple way: the given triangle must be rotated so that the altitude appears automatically. Given the following triangle, there are three possibilities:  - ENRICHMENT: - Have the pupil work with triangles whose altitudes can be located outside the triangle. - Have the pupil observe that in the equilateral triangle: - the altitudes and angle bisectors coincide, - the altitudes and angle bisectors are axes of symmetry of the triangle. - Have the pupil observe that the axis of symmetry of an isosceles triangle coincides with one of the altitudes and one of the angle bisectors of the triangle.

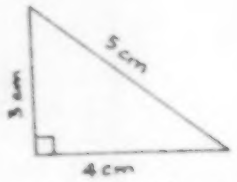
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.3.4 TO CONSTRUCT a quadrilateral, given sufficient data.	I	- In elementary school, the pupil has learned to "draw quadrilaterals... in which determined characteristics are known." - The pupil should be encouraged to construct his figures as accurately as possible. - The teacher should encourage the pupil to make use of a minimum amount of data. For example: Draw a quadrilateral whose: (a) opposite sides are parallel, (b) angles are right angles, (c) sides are congruent. - The data can be given in the form of instructions. For example: Construct a quadrilateral following the instructions below: (1) draw a vertical line segment of 2 cm, (2) make a 90° turn to the right, (3) draw a 5 cm line segment, (4) make a 90° turn downward, (5) draw a 2 cm line segment, (6) make a 90° turn to the left, (7) draw a 5 cm line segment. 

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Mapping on grids is another activity which can be used to achieve this objective. - By way of example, here is an activity which involves drawing a figure by mapping the distances specified in an ordered series, and by turning in the same direction each time a term is changed. This series should be repeated several times and observations noted. One such series could be 2, 3, 2, 3, 2, 3, 2, 3, ... which in notation form is (2,3). - The following is the mapping of the series (2,3) starting at point A: (a) on a square grid  (b) on another type of grid  Steps in Construction 1. The direction of rotation is established. 2. A vertical line segment of 2 units is drawn. 3. A 90° turn is made. 4. A line segment of 3 units is drawn. 1. The direction of rotation is established. 2. A line segment of 2 units is drawn in a north-easterly direction. 3. A 120° turn is made. 4. A line segment of 3 units is drawn.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		5. A 90° turn is made. 5. A 120° turn is made. 6. A line segment of 2 units is drawn. 6. A line segment of 2 units is drawn. 7. A turn ... 7. A turn ... If the instructions have been correctly understood, figures with different patterns can be drawn: e.g. (4,3,2,1), (1,3,5), (2,2,1), (1,3), (4), (1,2,3), (3,2,1), etc. - The pupil could use pieces of a meccano set to construct a square. He could then distort it and draw the new figures obtained. In a similar manner, he could make a rectangle, distort it, and draw the new figures obtained. Numerous other techniques could also be explored.
5.3.5 TO CONSTRUCT a regular polygon of 3, 4, 5, 6, 8, or 10 sides, given sufficient data.	I	- It is possible to construct all the regular polygons by means of rotations and reflections. For example: Construct an equilateral triangle using the line segment below and 2 rotations of 60°. 

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		For example: Use as few transformations as possible to construct a hexagon using the following line segment: 
		- The teacher should make sure that the pupil observes that the centre of a regular polygon is found at the point of intersection of the right bisectors of two adjacent sides of the polygon.
		- The following observation can be made: The image of a regular polygon obtained by a rotation of $\frac{360^\circ}{\text{number of sides}}$ about its centre coincides with the original figure.
		- The teacher must make sure that the pupil is not simply performing these constructions mechanically, but that he is thinking about what he is doing so that later he will be able to deduce the properties of the different polygons studied.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The data provided could include: - the length of a side and the measure of an interior angle, - the number of sides and the perimeter, - the number of sides and the area, - etc. - The pupil should be encouraged to construct his figures as accurately as possible.
5.3.6 TO CONSTRUCT the axis or axes of symmetry, given a line segment, an angle, a triangle, a circle, etc.	I	- In elementary school the pupil has learned to "identify and to construct the axes of symmetry in a figure or between two figures." - A line will be referred to as an axis of symmetry if it is a line over which a reflection image that coincides with the original figure can be produced. - The teacher should point out to the pupil that certain axes of symmetry correspond to known line segments within the figures being studied: - the axis of symmetry of an angle is called the angle bisector; - the axis of symmetry of an isosceles or equilateral triangle is called an altitude; - the axis of symmetry of a square which passes through opposite vertices is called a diagonal; - the axis of symmetry of a circle is called a diameter; - etc.  The regular pentagon has 5 axes of symmetry.  The regular hexagon has 6 axes of symmetry.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.3.7 TO CONSTRUCT a plane figure similar to a given plane figure (with or without a constant of proportionality, i.e. scale factor).	II	- These constructions can be done: - freehand, - using a ruler, a protractor, or a compass, - by folding paper, - using a mirror, - etc. - The degree of accuracy of these constructions will depend in large part on the instruments used. - Different techniques can be used to enlarge or reduce a given plane figure. - An algebraic method can be used: - each dimension of the figure to be enlarged is multiplied by the scale factor and the new figure is drawn according to the new values calculated. - The pupil must remember that the angles remain constant. - For example: Given a scale factor of $\frac{3}{2}$, ENLARGE the figure below.  original figure

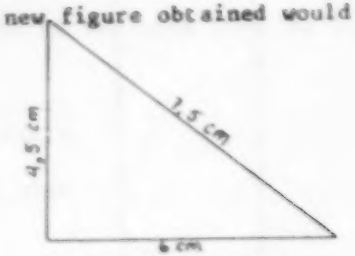
INTERMEDIATE OBJECTIVES	LEVELS	NOTES

$3\text{ cm} \times \frac{3}{2} = \frac{9}{2} = 4{,}5\text{ cm}$

$4\text{ cm} \times \frac{3}{2} = 6\text{ cm}$

$5\text{ cm} \times \frac{3}{2} = \frac{15}{2}\text{ cm} = 7{,}5\text{ cm}$

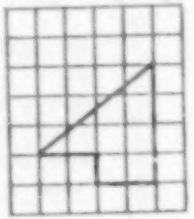
The new figure obtained would be:



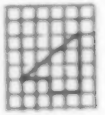
enlarged figure

- The use of square grids respecting the scale factor makes it possible to find the image of a given plane figure.

For example: Construct a plane figure by applying a scale factor of $\frac{1}{2}$ to the figure below.



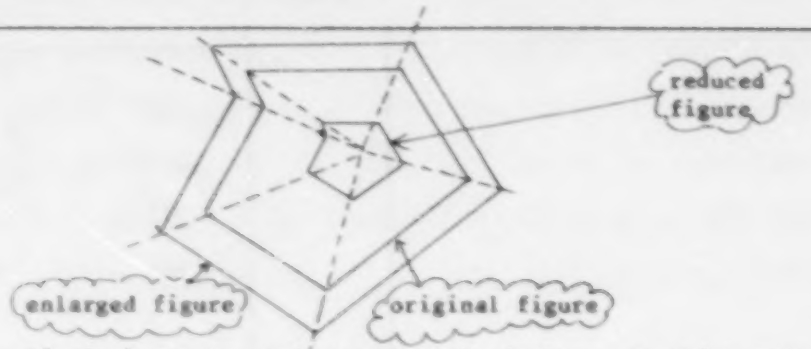
original figure



reduced figure

- The spider's web technique can also be used for enlarging or reducing a figure.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES



- The value 1(unity) can be used as the scale factor , in which case the similarity becomes a congruence.

- More concrete situations can also be used to attain this objective:

  - the drawing of plans for a building using a given scale;

  - the drawing to scale of a map of a given region;

  - using and interpreting a map;

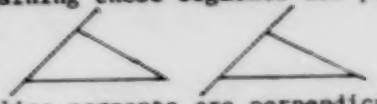
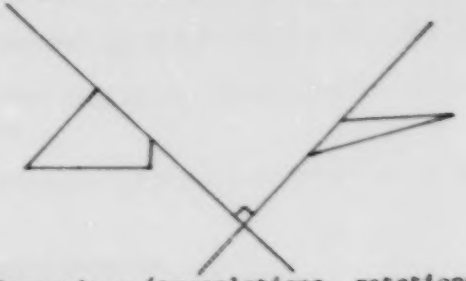
  - etc.

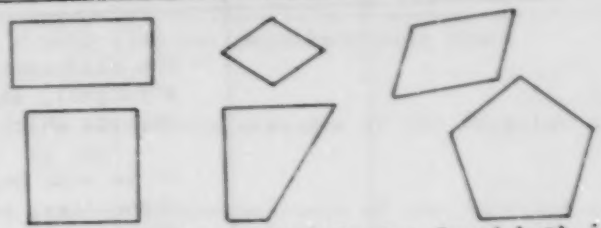
- In mathematics, the scale factor is represented by a ratio such as $\frac{3}{4}$. On a plan or map, this ratio is usually written 3:4, provided that the units of measurement are the same. The symbol $\hat{=}$, read as "correaponds to," is used if the units of measurement are different, in which case the units used must be indicated to comply with SI rules: 3 cm $\hat{=}$ 4 m.

- The scale factors used on plans or maps are often expressed in the form 1:10, 1:20, 1:100, 1:1000, 1:10 000, 1:100 000, etc.

TERMINAL OBJECTIVE 5.4 TO IDENTIFY the various properties of triangles and quadrilaterals.

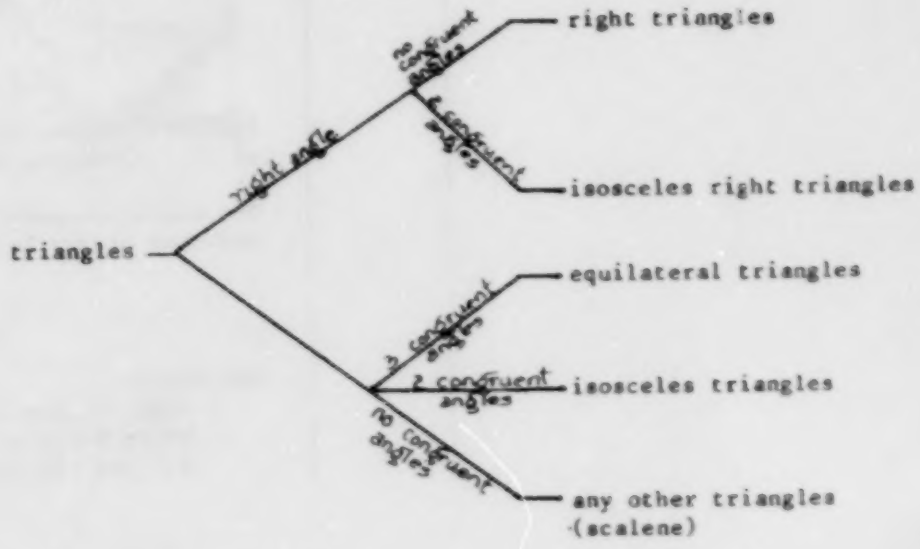
The geometric transformations presented in Terminal Objectives 5.1 and 5.2 can be useful in identifying the various properties of a given figure. By presenting the task of identifying these properties in terms of transformations, the teacher is able to integrate the different aspects of geometry in his teaching, and at the same time, to avoid the danger of covering this topic from a strictly technical point of view. It would therefore be advantageous to teach pupils how to construct geometric figures (Objective 5.3) and at the same time to identify their characteristic properties.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.4.1 TO IDENTIFY in certain figures, line segments which are parallel or perpendicular.	I	- In elementary school, the pupil has learned "to identify and describe parallel lines and perpendicular lines." - The pupil should study this objective through concrete examples before he attempts to identify these lines in the plane. - "Parallel line segments" or "perpendicular line segments" may be mentioned as long as it is clear that: - two line segments are parallel if and only if the lines containing these segments are parallel.  - two line segments are perpendicular if and only if the lines containing these segments are perpendicular.  - Geometric transformations (translations, rotations of 90°, reflections) can serve as extremely practical means of identifying parallel or perpendicular lines. - The line segments contained in a given figure could also be examined. For example: Identify the geometric figures in which the diagonals intersect each other at right angles. (See following diagrams.)

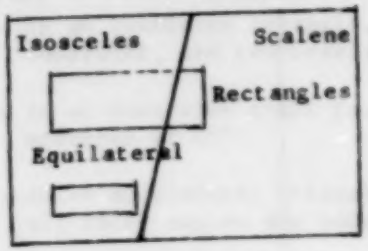
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.4.2 TO IDENTIFY in certain figures, congruent angles, complementary angles, supplementary angles, vertically opposite angles, acute angles, obtuse angles, right angles, etc.	I	 - The pupil should identify the altitudes found both inside and outside a triangle, a rectangle, a square, or a trapezoid. - In elementary school, the pupil has learned to identify right angles, acute angles, and obtuse angles. - Two definitions of an angle are given below: - "An angle is the union of two rays having the same endpoint." - "An angle is formed by the rotation of a ray about a point." - The first definition is static and satisfies the requirements of Euclidian geometry. The second definition is dynamic and well-suited to transformational geometry. - With regard to angle measure, - the first definition does not allow for a measure greater than 180°; - the second definition allows for angles of all measures. - The second definition is preferable particularly in the context of performing geometric transformations.

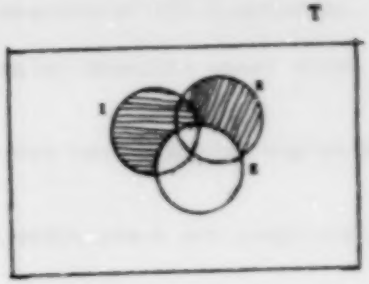
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The different types of angles, (zero, acute, right, obtuse, and straight), should be identified by using simple geometric figures or objects which are familiar to the pupil. - It is also hoped that the pupil will be able to identify adjacent, complementary, supplementary, and vertically opposite angles, using simple figures. - The pupil can identify congruent angles (not to be referred to as "equal angles") by: - using a protractor, - by superimposing, - by tracing, - etc. - The pupil can be asked to identify certain angles (alternate interior, alternate exterior, and corresponding angles) given the following situations: - two non-parallel lines cut by a transversal, - two parallel lines cut by a transversal. - It should be brought to the pupil's attention that if two parallel lines are cut by a transversal: - the alternate interior angles are congruent, - the alternate exterior angles are congruent, - the corresponding angles are congruent. - At this point the pupil should also be reminded of the definition of an angle bisector in terms of a rotation or a reflection.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.4.3 TO FIND the sum of the interior angles of a triangle and of a quadrilateral.	I	- The pupil should find out experimentally that: - the sum of the measures of the interior angles of a triangle is 180°, - the sum of the measures of the interior angles of a convex quadrilateral is 360°. - The teacher can help the pupil to understand the first proposition: - by having him measure the three angles of a triangle, - by having him use a translation. The first proposition often applies in geometry and trigonometry, and it is, therefore, important that the pupil remember it. - The second proposition follows from the first one, since any quadrilateral can always be divided into two triangles: - ENRICHMENT: - FIND the sum of the measures of the exterior angles of a triangle. - DETERMINE the relationship between an exterior angle of a triangle and the two non-adjacent interior angles.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.4.4 TO FIND the ratio of the measure of the side opposite the 30° angle to the measure of the hypotenuse in a right triangle.	II	- The pupil should find out experimentally that: In a right triangle, the measure of the side opposite a 30° angle is half the measure of the hypotenuse. - This proposition often applies in geometry and trigonometry and it is, therefore, important that the pupil remember it.
5.4.5 TO CLASSIFY triangles according to the measures of their angles and of their sides.	I	- This objective was presented to the pupil in elementary school: "To classify triangles according to the properties of their angles and of their sides." - The following diagram makes it possible to visualize such a classification: 

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- ENRICHMENT: -Classify the different types of triangles using a diagram.





Key:

T: triangles
R: right triangles
I: isosceles triangles
E: equilateral triangles

- FIND the point of intersection of the three medians of a triangle.

  - The pupil should find out experimentally that:

    - the three medians of a triangle meet at a single point.

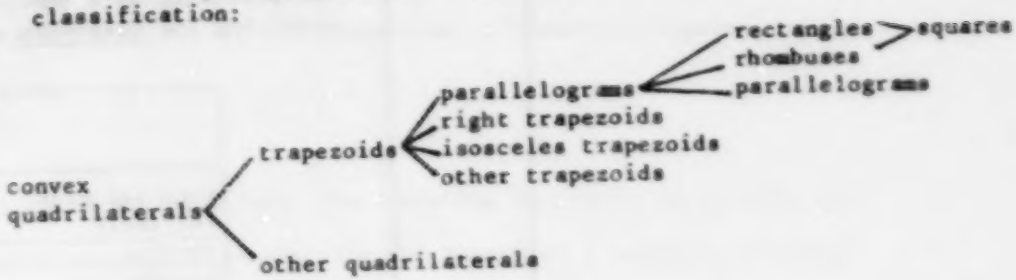
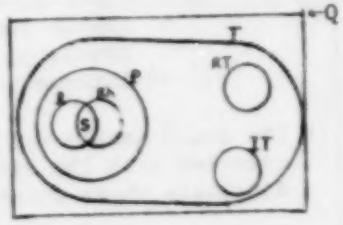
  - The pupil should find out experimentally that this point of intersection makes it possible to locate the centroid of the triangle under study. (Certain minimum conditions must, of course, be observed during this experiment.)

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- FIND the properties of an altitude, a median, a perpendicular bisector, and an angle bisector drawn from a given vertex of an equilateral or an isosceles triangle. - The pupil should find out experimentally that: - in an equilateral triangle, the altitude, the median, the perpendicular bisector, and the angle bisector drawn from a given vertex coincide. - in an isosceles triangle, the altitude, the median, the perpendicular bisector, and the angle bisector drawn from the vertex whose measure is different from that of the other two angles, coincide.
		- FIND the properties of the sides and angles of right triangles, isosceles triangles, isosceles right triangles, and equilateral triangles. - The main properties for the pupil to discover and remember are listed below: - The sum of the measures of the interior angles of any triangle is 180°. - In a right triangle, the square of the measure of the hypotenuse is equal to the sum of the squares of the measures of the other two sides.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- In a right triangle, the measure of the side opposite a 30° angle is equal to half the measure of the hypotenuse. - In an isosceles triangle, the angles opposite equal sides are congruent, and conversely. - In an isosceles right triangle, the two congruent angles have measures of 45°. - In an equilateral triangle, all three sides are congruent and all three angles are congruent. - In an equilateral triangle, the congruent angles have measures of 60°. - In an isosceles right triangle, the measure of the hypotenuse is equal to $\sqrt{2}$ times the measure of one of the sides about the right angle. - etc.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.4.6 TO CLASSIFY convex quadrilaterals according to their angles, their sides, and their diagonals.	I	- The following is a summary chart of what the pupil can find out by examining quadrilaterals:

	Parallelism	Sides	Angle Measures	Diagonals
Parallelogram	opposite sides parallel	opposite sides congruent	opposite angles congruent	bisect each other
Rectangle	opposite sides parallel	opposite sides congruent	90°	bisect each other and are congruent
Rhombus	opposite sides parallel	all sides congruent	opposite angles congruent	bisect each other at right angles
Square	opposite sides parallel	all sides congruent	90°	bisect each other at right angles and are congruent
Scalene Trapezoid	2 sides parallel	no particular properties	no particular properties	no particular properties
Right Trapezoid	2 sides parallel	no particular properties	2 adjacent angles of 90°	no particular properties
Isosceles Trapezoid	2 sides parallel	non-parallel sides congruent	angles adjacent to the same base are congruent	congruent

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- This objective has been presented to the pupil in elementary school: "To classify quadrilaterals according to the properties of their angles and of their sides." - The following diagram makes it possible to visualize such a classification:  - The teacher should help the pupil find out the relationships among the different types of quadrilaterals by means of a Venn diagram:  Key: Q= quadrilateral P= parallelogram R= rectangle Rh= rhombus S= square T= trapezoid RT= right trapezoid IT= isosceles trapezoid This activity will enable him to form a better conception of the relationships between the different categories of quadrilaterals.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES

- A diagonal should be defined as a line segment which joins two non-consecutive vertices of a quadrilateral.

- The following is a summary chart of what the pupil can find out by examining the diagonals of convex quadrilaterals.

	middle	90°	congruent diagonals
parallelogram	yes	no	no
rectangle	yes	no	yes
rhombus	yes	yes	no
square	yes	yes	yes
scalene trapezoid	no	yes	no
right trapezoid	no	yes	no
isosceles trapezoid	no	yes no	yes

* divide the figure into 2 congruent triangles* [bisect the area].
** intersect at right angles.

- The pupil should find out which quadrilaterals have images that coincide with the original figure after a rotation of 180° and have diagonals whose point of intersection is the centre.

- The pupil should find out which quadrilaterals have diagonals that are axes of symmetry.

- The pupil should find out in which quadrilaterals the sum of the measures of two consecutive interior angles is equal to 180°.

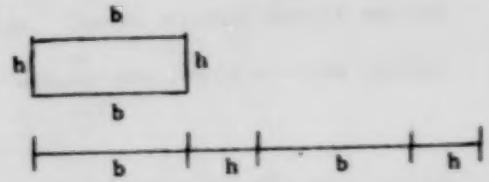
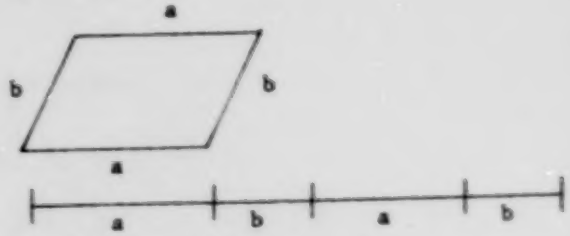
- ENRICHMENT: - Doing tesselations can provide an interesting source of enrichment for the pupil.

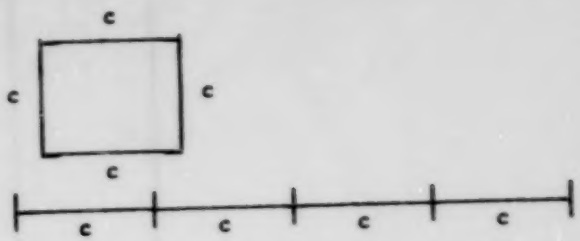
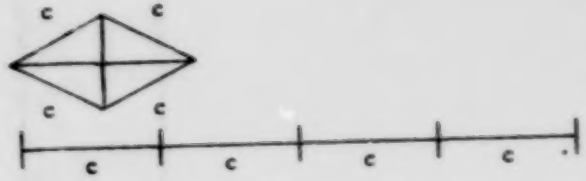
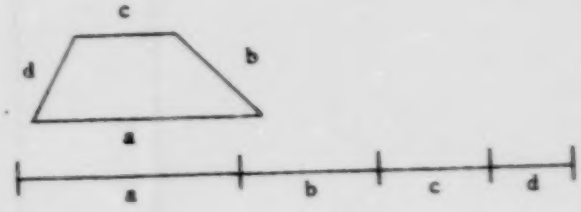
TERMINAL OBJECTIVE 5.5 TO SOLVE problems of everyday life using units of length or of area.

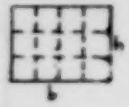
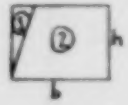
The aim of this objective is to develop in the pupil the skill to obtain a certain amount of numerical information from situations in everyday life, using direct or indirect measurement. It is also intended to enable him to find out other information which is not immediately evident, by means of the logical application of geometric transformations, of the properties of geometric figures, of the formulas for perimeter and area, and so on.

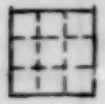
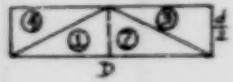
Measurement plays an important part in human activity and, like locating an object on a line, can become second nature to us. It is used primarily in determining the length of a segment, in calculating surface area, and so on.

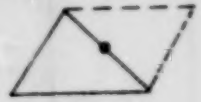
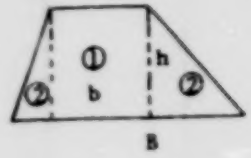
Note: In the following pages, the algebraic transformations appearing with the transformational objectives are offered as suggestions only.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.5.1 TO CALCULATE the perimeter or the area of trapezoids.	I	- A trapezoid is a quadrilateral with a pair of parallel sides. - From this definition it follows that a parallelogram, a rhombus, a rectangle, and a square are all trapezoids, since each one has at least one pair of parallel sides. - The perimeter or area of a trapezoid can be calculated using formulas which the teacher must ensure that the pupil learns. - The formulas for calculating perimeter can be determined easily by unfolding the following figures: - <u>rectangle</u>:  $p = b + h + b + h$ $p = 2b + 2h$ $p = 2(b + h)$ - <u>parallelogram</u>:  $p = a + b + a + b$ $p = 2a + 2b$ $p = 2(a + b)$

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- <u>square</u>:  $p = c + c + c + c$ $p = 4c$
		- <u>rhombus</u>:  $p = c + c + c + c$ $p = 4c$
		- <u>trapezoid</u>:  $p = a + b + c + d$

INTERMEDIATE OBJECTIVES	LEVEL	NOTES
		- The formulas used to find area are in some cases less apparent. Certain geometric transformations can be used to find these areas. - <u>rectangle</u>:  * Taking the square ■ as the basic unit, then there are b square units in each of h rows, hence b x h square units in all: A = b x h - <u>parallelogram</u>: Translating the triangle gives:  And since translations preserve area: A = b x h

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- <u>square</u>:  Since a square is a type of rectangle, it is possible to use the formula: $A = b \times h$ or: $b = c$ and $h = c$ hence $A = c \times c$ $A = c^2$
		- <u>rhombus</u>: By appropriately translating each triangle below the long diagonal D, it is possible to obtain:  and since translations preserve length, all that is required is to calculate the area of the rectangle: $A = D \cdot \frac{d}{2}$ hence, $A = \frac{Dd}{2}$.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Before the pupil attempts to find the formula for the area of a trapezoid, it is necessary for him to know how to find the area of a triangle. Consider a parallelogram: A diagonal divides a parallelogram into two triangles which, if examined, can be seen to be congruent. In fact, a rotation of 180° with its center at the midpoint of the diagonal matches triangle 1 onto triangle 2.  Since the formula for the area of a parallelogram is: $A = bh$, it follows that the area of each triangle is: $A = \frac{b \cdot h}{2}$. - <u>trapezoid</u>:  By translating each triangle, the following two figures can be obtained:

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		And since translations preserve length, it is possible to find the area of the trapezoid by finding the area of the rectangle and of the triangle.

$$A_1 = bh \qquad A_2 = \frac{(B - b)h}{2}$$

hence,

$$A = A_1 + A_2$$

$$A = bh + \frac{(B - b)h}{2}$$

$$A = \frac{2bh + (B - b)h}{2}$$

$$A = \frac{h(2b + B - b)}{2}$$

$$A = \frac{h(B + b)}{2}$$

$$A = h\,\frac{(B + b)}{2}.$$

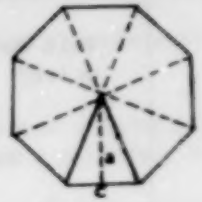
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.5.2 TO CALCULATE the circumference or the area of a circle, given the radius or the diameter.	II	- The pupil should be encouraged to discover the formula for the circumference of a circle. - One possible approach consists of measuring the circumference and the diameter of several circular objects and of calculating the quotient $\frac{c}{d}$ or $\frac{c}{2r}$. The following table can then be completed:

	c	d	$\frac{c}{d}$
1			
2			
3			
4			
5			
6			
7			
.			
.			
.			

If the measurements taken are sufficiently accurate, the pupil should observe that the values of $\frac{c}{d}$ appears to remain constant.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		This constant is designated by the letter π. It is thus possible to write: $\frac{c}{d} = \pi$ hence, $c = \pi d$ or, $c = \pi . 2r$ $c = 2\pi r.$ - If the pupil is not using a calculator, he should use an approximate value for π: $\pi \approx 3,1416$ $\pi \approx \frac{22}{7}.$ - It should be brought to the pupil's attention that π is an infinite non-repeating decimal: $\pi = 3,141\ 592\ 653 \ldots$ and that it cannot be said that π equals 3,1416 or that π equals $\frac{22}{7}$, since these values are only approximations. - The formula for the area of a circle can be derived from the fact that a circle can be considered as a regular polygon with a large number of sides.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		The area of a regular polygon can be calculated as follows:



$$A = n \frac{(a \cdot c)}{2}$$

$$A = n \frac{(c \cdot a)}{2}$$

$$A = \frac{n \cdot c \cdot a}{2}$$

but, $p = n \cdot c$

hence, $A = \frac{p \cdot a}{2}$.

- If the circle is regarded as a regular polygon with a large number of sides, it is possible to state that:

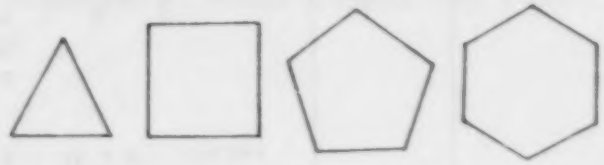
$$p = C = 2\pi r$$

$$a = r$$

and thus, by substituting into $A = \frac{pa}{2}$, it is possible to obtain:

$$A = \frac{(2\pi r)r}{2}$$

$$A = \pi r^2.$$

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- ENRICHMENT: - Given the circumference of a circle, calculate its area.
5.5.3 TO CALCULATE the circumference of a circle, given the measure of a central angle and of the intercepted arc.	II	- This objective is not a question of presenting the formula generally used to solve such a problem, but one of establishing a ratio: If the angle θ intercepts an arc s of a circle, then the circumference c is to the arc s as $360°$ is to the measure of angle θ. $\frac{c}{s} = \frac{360}{\theta}$. - ENRICHMENT: - Have the pupil derive the formula $s = \frac{2\pi r\theta}{360}$ from the preceding ratio.
5.5.4 TO CALCULATE the perimeter of a polygon by decomposing or transposing it into simpler figures.	I	- In the case of regular polygons, the perimeter can be calculated using the formula $p = nc$ where p stands for the perimeter of a regular polygon, n stands for the number of sides in the polygon, c stands for the length of a side. The pupil should complete the following table using the dimensions of certain regular polygons: 

INTERMEDIATE OBJECTIVES	LEVELS	NOTES

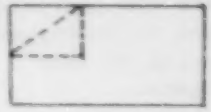
n	c	p	n . c
3			
4			
5			
6			
.	.	.	.
.	.	.	.
.	.	.	.

The pupil should note the similarity between the entries in the last two columns in the table and should be able to deduce that:

$$p = nc.$$

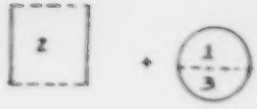
- Some possible examples of polygons are:

(a)

which becomes (by reflection)


hence: $p = 2(b + h)$.

(b)
which becomes (by reflection)

which becomes (by transformation)


hence: $p = 2c + 2\pi c$

$$p = 2c(1 + \pi).$$

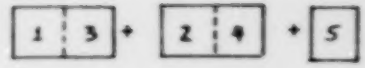
INTERMEDIATE OBJECTIVES	LEVELS	NOTES

(c)

which becomes (by translation)

which becomes (by translation)

which becomes (by transformation)

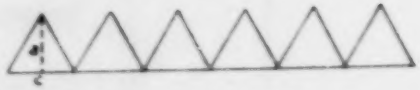


hence: $p = 2(2c + c) + 2(2c + c) + 4c$

$p = 6c + 6c + 4c$

$p = 16c.$

- It would be useful for the pupil to be able to use geometric transformations to change polygons into simpler figures.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.5.5 TO CALCULATE the area of a polygon by decomposing or transforming it into simpler figures.	II	- In the case of regular polygons, the area can be calculated using the formula: $A = \frac{pa}{2}$ where: A stands for the area of a regular polygon, a stands for the altitude of one of the triangles (the apothem), p stands for the perimeter of the regular polygon. The pupil should divide a regular polygon into triangles. The following polygon can be divided as follows:  The area of the small triangle is given by: $A_1 = \frac{ac}{2}$ and the total area is: $A = nA_1$ $A = n \cdot \frac{ac}{2}$

INTERMEDIATE OBJECTIVES	LEVELS	NOTES

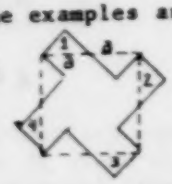
$$A = \frac{nac}{2}$$

$$A = \frac{nca}{2}$$

$$A = \frac{pa}{2}.$$

- In the case of polygons which are not regular, it is helpful to use geometric transformations to change the polygon into a simpler figure.

Some examples are:

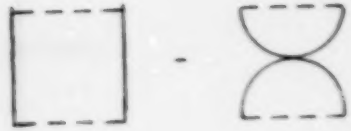
(a)  which becomes (by rotation)

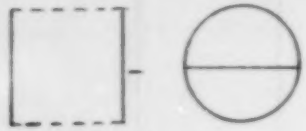
hence: $A = c^2$

$A = (a + a)^2$

$A = (2a)^2$

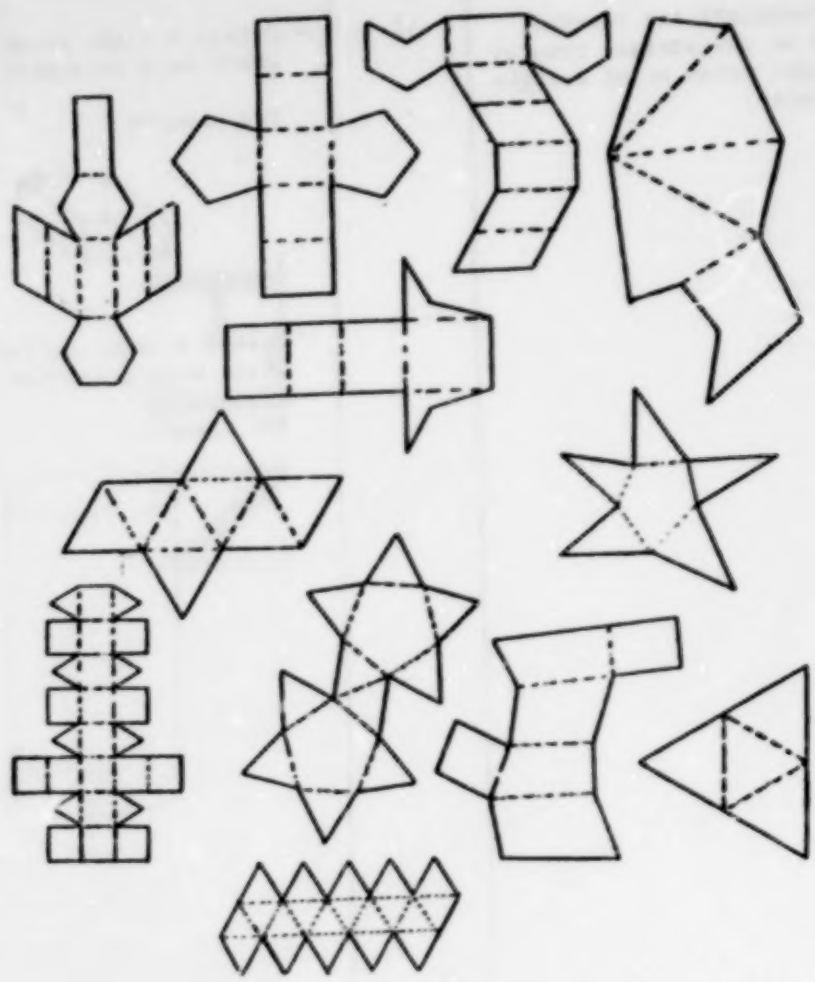
$A = 4a^2$.

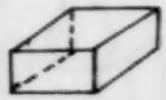
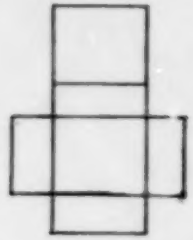
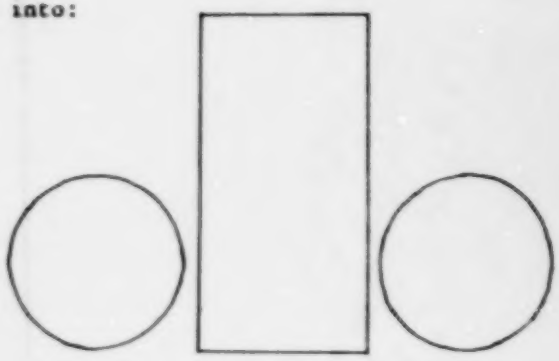
(b) which becomes (by transformation) 

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		or 
		hence: $A = (c^2) - \pi\left(\frac{c}{2}\right)^2$
		$A = c^2 - \pi\frac{c^2}{4}$
		$A = \frac{4c^2 - \pi c^2}{4}$
		$A = \frac{c^2\,(4 - \pi)}{4}.$
5.5.6 TO CALCULATE the measure of one dimension of a plane geometric figure, given its perimeter or area, and the pertinent dimensions.	II	- The pupil should be made aware that the algorithms for solving equations will be extremely useful to him in geometry as well. - The term "pertinent dimensions" refers to either: - the number of sides in a regular polygon; - the measure of one side of a rectangle; - the altitude or base of a triangle; - etc. - At this point, it would be useful for the pupil to investigate the set of polygons which have the same perimeter or the same area.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		All the triangles below have the same area: The figures below can be shown to have the same perimeter:
5.5.7 TO RECONSTRUCT a solid, given its component plane geometric figures.	II	- In elementary school, the pupil has learned "to construct solids by means of plane figures whether identical or not" and "to investigate the characteristics of different solids." - The aim of the present objective is to develop the pupil's powers of observation. - The pupil should therefore learn to construct a right prism in the form of: - a cube, - a rectangular parallelepiped, - a cylinder, - etc.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- The pupil should find out experimentally that by rearranging certain figures appropriately it is possible to construct a solid. For example: Four isosceles triangles and a square make it possible to construct a pyramid. The process is as follows: - The pupil should be able to construct these solids with a reasonable degree of accuracy: - in space using boxes; - on paper by conveying perspective. - Some of the plane geometric figures which make it possible to construct certain solids are shown on the following page.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
5.5.8 TO CALCULATE the lateral area or the surface area of a right prism or of a right cylinder.	II	- Unfold a right prism into simple figures using the formulas for area which were presented in Objective 5.5.1. For example:  can be unfolded into:  - Unfold a right cylinder into simple figures using the formulas for area which were presented in Objective 5.5.1. For example: can be unfolded into: 

GENERAL OBJECTIVE

6

TO INTRODUCE the pupil to the language and to the application of elementary statistics.

THEME: Statistics

TIME: 6% in Secondary I
9% in Secondary II

Statistical tables and graphs are in common use today. They often appear on television, in newspapers, in magazines of both general and specific interest, and in other places. The pupil should be able to evaluate objectively the information presented in them and the conclusions which can be drawn from them.

The main purpose of this general objective is to develop the pupil's capacity for objective evaluation so that he will become warier of the many ways in which statistical tables and graphs are interpreted by the media. For this reason, an important part of this program has been devoted to statistics. Through the study of statistics in Secondary I, the pupil learns to obtain information from different types of graphs illustrating familiar situations and to draw up tables of statistical data drawn from these situations. In Secondary II, he learns to construct a number of different kinds of graphs (broken-line, bar and circular) in order to illustrate familiar situations taken from history, geography, and science. He is thereby better prepared to formulate his own opinions as to the validity of surveys and is less likely to be misled by his own lack of knowledge.

The terminal objectives in this section can be attained by having the pupil investigate a topic of his choice. The pupil can begin his investigation by doing background research on the topic. This will enable him to attain Terminal Objective 6.1 before he conducts the investigation. The actual illustration in the form of tables of the data he has collected during his investigation constitutes the last stage in this program in the first cycle of secondary school.

At the end of this section it would be appropriate to return to Objective 6.1 for two reasons: to obtain from the different graphs which have been drawn the information pertinent to the purpose of the investigation, and to choose subjects for enrichment activities which would enable the pupil to practice and refine different methods of investigation.

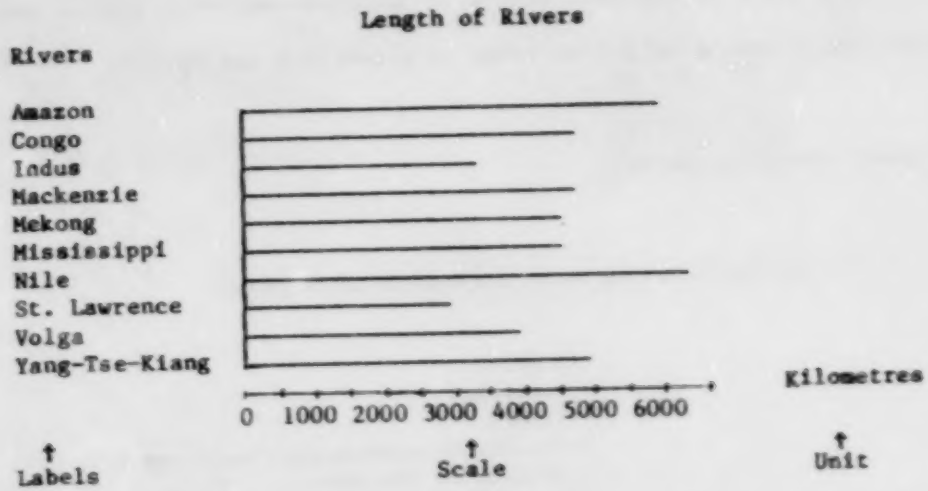
Investigation is not the only activity which lends itself to the study of statistics. It is also possible to have the pupil do the following: to study the information indicated on a disk by a needle whose position is determined by chance, to read data from a table, or to compile data on the basis of probability experiments such as the rolling of dice or the choosing of marbles from a set of marbles of assorted colours.

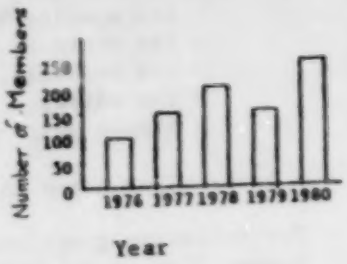
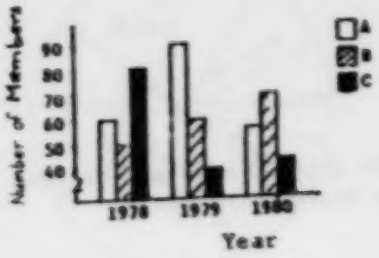
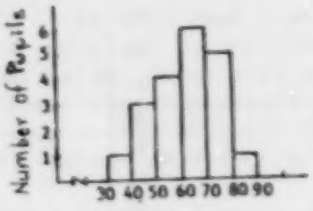
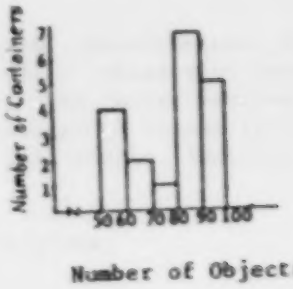
TERMINAL OBJECTIVE 6.1 TO COLLECT information from different kinds of graphs illustrating familiar situations.

If this objective is to be achieved, it is important for the teacher to begin with situations which the pupil is likely to encounter in his own life. A prerequisite activity which would help the pupil appreciate the value of his mathematics course would be to have him look for graphs in newspapers and magazines. The pooling of these graphs will provide him with an abundant source of material and will enable him to see that statistics can help him to achieve a better understanding of a wide range of situations and events.

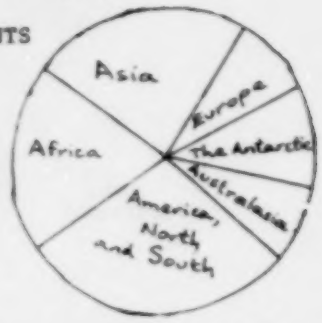
The pupil should be able:

- to identify the main components of a graph:
  - title,
  - headings,
  - scale,
  - labels or subheadings of items,
  - breaks on the axes,
  - legend;
- to compare certain items on a graph;
- to describe the general situation represented by the graph.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
6.1.1 TO COLLECT information from a bar graph.	I	- In order for a graph to be read, it is essential that the data items be labelled, that a basic unit of measurement be indicated, and that the scale be marked off in rounded figures on the axes.  - Bar or rectangular graphs (with or without spaces between the bars) usually consist of between 5 and 10 bars whose width is of no consequence.

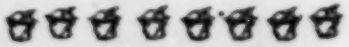
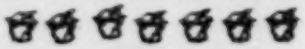
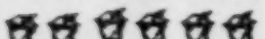
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Some other graphs are shown below: (a) Sales Record of Troop C  (b) Sales of Categories A, B, and C  (c) Test Scores  (d) Number of Containers in Relation to Number of Objects  In examples (a), (b), (c), and (d) quantities are clearly marked on the vertical axis, while labels for different items are shown on the horizontal axis.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- As the pupil studies the graph, he should observe: - the width of the bars, - whether they are placed vertically or horizontally, - the spacing between them, - the scale or graded intervals, - the headings, - the title, - breaks which may or may not appear on the axes.
6.1.2 TO COLLECT information from a broken-line graph.	I	- A broken-line graph is used to represent a continuous phenomenon. - The following graph represents the changes in temperature at Quebec City on July 20 of year X. TEMPERATURE CHANGES IN QUEBEC CITY JULY 20 19XX

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
6.1.3 TO COLLECT information from a circular graph.	I	- The following is an example of a circular graph: AREAS OF THE CONTINENTS  - Given the data items and the number of degrees in each sector, it is possible to determine what percentage of the total area each continent represents.
6.1.4 TO COLLECT information from a pictogram.	I	- It is easy to read a pictograph or pictogram. Nevertheless, it is important to note that there are two types: those with repeated motifs and those with proportional motifs. The latter involve three-dimensional representations. Since the concept of volume is less familiar to the pupil than that of area, only simple, familiar situations should be presented. - The following figures are examples of pictograms. - With repeated motifs:

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		(a) Population of a city (by age groups):

POPULATION OF CITY X BY AGE GROUP

0 - 14	
15 - 29	
30 - 44	
45 - 49	
60 and over	

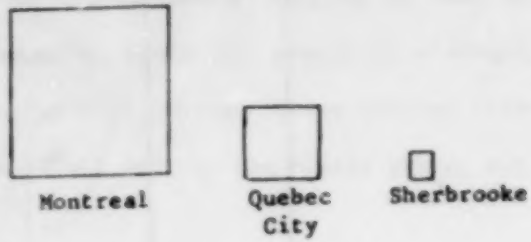
≙ 100 people

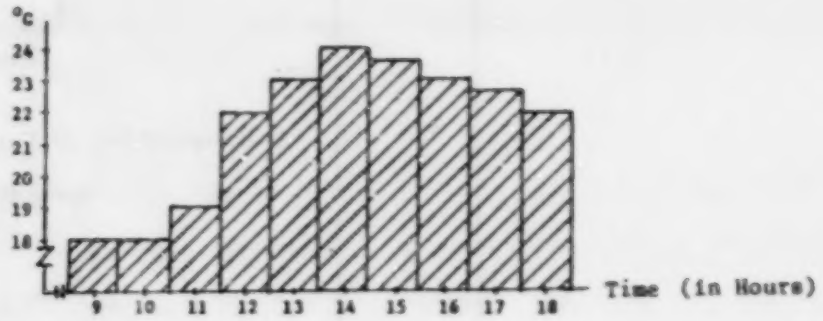
(b) Means of transportation used by pupils to go to school:

MEANS OF TRANSPORTATION

Car	
School bus	
City bus	
Bicycle	
Foot	

≙ 10 people

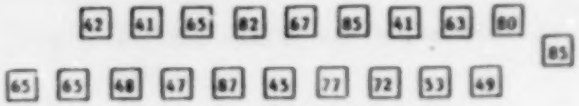
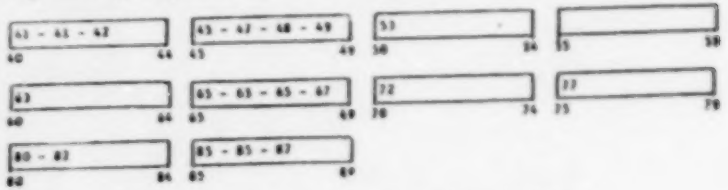
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- with proportional motifs: (a) Comparative size of three cities:  (b) Population of Latin America
6.1.5 TO COLLECT information from a histogram.	I	- A histogram, like a broken-line graph, is used to represent continuous data. - The temperature changes shown in Objective 6.1.2 could be represented by the following histogram:

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		TEMPERATURE CHANGES JULY 10 19XX IN QUEBEC CITY 

TERMINAL OBJECTIVE 6.2 TO CONSTRUCT a table of statistical data drawn from familiar situations.

Conducting an investigation constitutes a useful preliminary step to the construction of statistical tables because it enables the pupil to collect from situations which are familiar to him the data needed to construct such tables. At this point the teacher should stress the importance of a good choice of samples so that sufficient data is gathered to represent adequately the whole being studied. For example, would the pupils in a single classroom be an adequate sample for an investigation whose purpose is to determine the most popular Monday evening television show among pupils in the school, or, would it be preferable to include pupils of all ages in the sample group; are the questions in the opinion poll biased, and so on?

Once the pupil has gathered the necessary information, he should compile it in such a way that later analysis of the data will be easy: this is the step in which he constructs data tables. In the first cycle of secondary school, the pupil is restricted to tabulating the different categories of data in different ways and to computing their frequencies.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
6.2.1 TO COMPILE data.	I	- The pupil should realize that the study of familiar situations often requires so much work that it is preferable to reduce his investigation to more manageable proportions. - He should choose his sample according to very specific criteria: - it must be representative of the entire group of elements with which the investigation is concerned; - it must include a sufficient number of elements selected at random from the reference group. - If the data is randomly selected, it must then be rearranged in some specific order. Here are the prices of a group of different articles:  Rearranged in order of price they are: If they are grouped into 10 classes, the result would be:  It is then possible to construct a data table.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
6.2.2 TO ENTER data in a table.	I	- It is very important that the pupil be required to construct his data tables in a clear and logical way, as these tables are an important step prior to the use and interpretation of the data for statistical purposes. - A table should include: - a title or name, - headings for each of the columns, - labels for the data items whether grouped or not. - Here is an example: Each time a piece of data is identified, a tally mark should be made on the table. When there is a group of 5, the fifth mark should be drawn obliquely through the other 4. This process continues until all the data has been tallied. NUMBER OF MARBLES WON Name / Frequency Frank — 𝍸 𝍸 𝍸 III Julie — 𝍸 𝍸 Oliver — 𝍸 III Stephanie — 𝍸 𝍸 III - This technique for tabulating frequencies in a table can be used with data which is grouped or not.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES

- Here is another example:

MATHEMATICS TEST SCORES

98, 26, 77, 58, 92, 27, 78, 48, 89, 32,
70, 71, 70, 60, 64, 71, 77, 74, 69, 67,

89, 97, 84, 79, 88, 35, 69, 91, 59, 94,
59, 75, 34, 76, 75, 48, 66, 73, 73, 66,
57, 59, 69, 75, 69, 67, 62, 62, 62, 64

41, 42, 47, 49, 50, 55, 43, 42, 48, 43,
65, 67, 65, 69, 68, 67, 63, 62, 81, 83,
85, 87, 86, 84, 88, 92, 97, 65, 68, 67,

77, 78, 71, 71, 72, 35, 78, 76, 81, 67,
85, 84, 83, 82, 72, 75, 86, 87, 81, 81.

Frequencies can be determined using intervals or classes. As a general rule, 7 to 10 classes are considered sufficient. The range of scores is 72 marks, that is, from 26 to 98. Intervals of ten can therefore be used.

MATHEMATICS TEST SCORES

Class Interval	Number of Pupils	Class Interval	Number of Pupils
20-29	11	60-69	~~++++~~ ~~++++~~ ~~++++~~ ~~++++~~ ~~++++~~
30-39	11	70-79	~~++++~~ ~~++++~~ ~~++++~~ ~~++++~~ ~~++++~~ 1
40-49	~~++++~~ ~~++++~~	80-89	~~++++~~ ~~++++~~ ~~++++~~ ~~++++~~ 1
50-59	~~++++~~ 11	90-99	~~++++~~ 11

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
6.2.3 TO CALCULATE frequencies.	I	- Any table of statistical data has two columns (or rows), one listing the data items, and the other giving the number of elements. - A good way of calculating frequencies is to construct a table. This method of tallying data makes it easy to compute frequencies quickly. Here are two examples: - The tables in 6.2.2 make it possible to construct the following table:

NUMBER OF MARBLES WON

Name	Frequency
Frank	19
Julie	10
Oliver	8
Stephanie	13

MATHEMATICS TEST SCORES

Class Interval	Frequency	Class Interval	Frequency
20-29	2	60-69	25
30-39	2	70-79	26
40-49	10	80-89	21
50-59	7	90-99	7

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
6.2.4 TO CONSTRUCT a table of statistical data from a bar graph.	I	- The main elements of a bar graph are: - the title or name of the graph; - the identification (labelling) of the data, grouped or not, on one of the axes; - the division of the scale on the other axis (the one showing frequencies), taking into account the highest frequency; - the bars, which are usually separate rectangles placed either vertically or horizontally. - The pupil should be able to enter in a table the different information obtained from the bar graph. - In Secondary II, the pupil will do the reverse, constructing a bar graph from a table of data. - This objective serves as a bridge between what the pupil has learned previously and what he will be asked to do in Secondary II.

TERMINAL OBJECTIVE 6.3 TO CONSTRUCT broken-line graphs or bar graphs to illustrate certain familiar situations.

In Secondary I, the pupil learned to collect information from different types of graphs. He is now at the stage at which he can be asked to construct such graphs in order to illustrate familiar situations.

Three distinct steps can be followed here:

1. conduct an investigation;
2. make tables;
3. make graphs;

The notes on Intermediate Objective 6.2 briefly summarize the nature of the investigation. It is important that the pupil be encouraged to carry out the investigation with a certain degree of accuracy. He must be made aware that it is this accuracy which determines the value that can be placed on the interpretation given to the tables and graphs used to present the results of the investigation. In this way, he will develop a critical attitude toward graphs and toward the ways in which they are often interpreted by the information media.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
6.3.1 TO CONSTRUCT a broken-line graph from a table of data.	II	- It is important for the pupil to remember that broken-line graphs usually represent continuous phenomena. - The pupil may construct his graph in two ways: - from a table of data which is provided, or - from a table of data which he has constructed himself. - The main elements of a broken-line graph are: - the title or name of the graph; - the identification (labelling) of the data, grouped or not, on the horizontal axis; - the division of the scale on the frequency axis (the vertical axis), taking into account the highest frequency; - the broken line. - It is important that pupils be required to present data carefully and to label the broken-line graph accurately. Pupils tend not to regard the title as an integral part of the graph, so they must be made aware that the title is, in fact, frequently essential for an accurate interpretation of the diagram. It is equally important that they identify the two axes of the graph accurately.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES

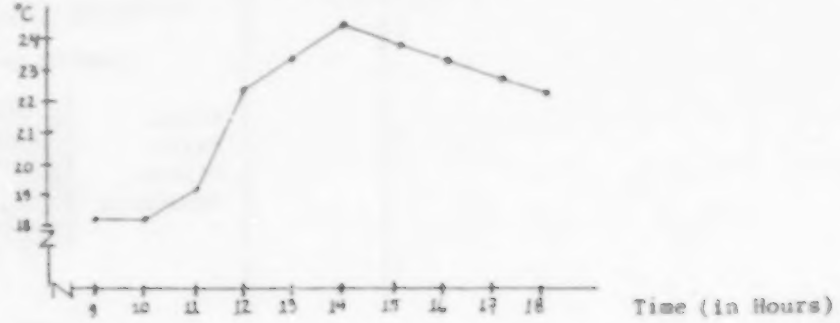
Here is an example:

TEMPERATURE CHANGES
ON JULY 20, 19XX
IN QUEBEC CITY

TIME	TEMPERATURE
9	18°C
10	18°C
11	19°C
12	22°C
13	23°C
14	24°C
15	23,5°C
16	23°C
17	22,6°C
18	22°C

and its representation in the form of a broken-line graph would be:

TEMPERATURE CHANGES
ON JULY 20, 19XX
IN QUEBEC CITY



INTERMEDIATE OBJECTIVES	LEVELS	NOTES
6.3.2 TO CONSTRUCT a bar graph from a table of data.	II	- The pupil may construct his graph in two ways: - from a table of data which is provided, or - from a table of data which he has constructed himself. - The main elements of a bar graph are: - the title or name of the graph; - the identification (labelling) of the data, grouped or not, on the horizontal axis; - the division of the scale on the frequency axis, taking into account the highest frequency; - the rectangular bars, usually spaced apart and placed vertically or horizontally. - It is important that pupils be required to present data carefully and to label the bar graph accurately. Pupils tend not to regard the title as an integral part of the graph, so they must be made aware that the title is, in fact, frequently essential to a sound interpretation of the diagram. It is equally important that they identify the two axes of the graph accurately. - The first table presented in Objective 6.2.3 makes it possible to construct the following graphs: NUMBER OF MARBLES WON Frank Julie Oliver Stephanie 5 10 15 20 NUMBER OF MARBLES WON 20 15 10 5 Frank Julie Oliver Stephanie

TERMINAL OBJECTIVE 6.4 TO CONSTRUCT a circular graph to illustrate certain familiar situations.

In Secondary I, the pupil learned to collect information from different types of graphs. He is now at the stage at which he can be asked to construct such graphs in order to illustrate familiar situations.

Three separate steps can be followed here:

1. conduct an investigation;
2. make tables;
3. make diagrams.

The notes on Intermediate Objective 6.2 briefly summarize the nature of the investigation. It is important that the pupil be encouraged to carry out the investigation with a certain degree of accuracy. He must be made aware that it is this accuracy which determines the value that can be placed on the interpretation given to the tables and graphs used to present the results of the investigation. In this way, he will develop a critical attitude toward graphs and toward the ways in which they are often interpreted by the information media.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
6.4.1 TO ESTABLISH a ratio comparing data for a given item to the total data.	II	- This step is essential for the construction of a circle graph. - The term "relative frequency" refers to the frequency expressed in terms of a fraction or per cent of the total number of items.
6.4.2 TO DETERMINE the measure of a central angle of a circular graph corresponding to the ratio established by comparing data of a given item to the total data.	II	- Using the ratios established in objective 6.4.1, we get:

NUMBER OF MARBLES WON

Name	Frequency	Relative Frequency
Frank	19	$\frac{19}{50}$
Julie	10	$\frac{10}{50}$
Oliver	8	$\frac{8}{50}$
Stephanie	13	$\frac{13}{50}$

$$360° \times \frac{19}{50} = 136,8°$$

$$360° \times \frac{10}{50} = 72,0°$$

$$360° \times \frac{8}{50} = 57,6°$$

$$360° \times \frac{13}{50} = 93,6°$$

It should be pointed out to the pupil that in this example:

$$136,8° + 72,0° + 57,6° + 93,6°$$

equals 360°.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- All that is required here is to add another column to the table presented in Objective 6.4.1.

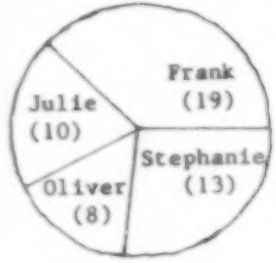
NUMBER OF MARBLES WON

Name	Frequency	Relative Frequency	Degree
Frank	19	$\frac{19}{50}$	136,8°
Julie	10	$\frac{10}{50}$	72,0°
Oliver	8	$\frac{8}{50}$	57,6°
Stephanie	13	$\frac{13}{50}$	93,6°
TOTAL	50	$\frac{50}{50}$	360°

6.4.3 TO DETERMINE the percentage corresponding to the ratio established by comparing data of a given item to the total data.	II	- This information often appears within the sectors of the circular graph to illustrate the information more clearly.

NUMBER OF MARBLES WON

Name	Frequency	Relative Frequency		Degree
Frank	19	$\frac{19}{50}$	38%	136,8°
Julie	10	$\frac{10}{50}$	20%	72,0°
Oliver	8	$\frac{8}{50}$	16%	57,6°
Stephanie	13	$\frac{13}{50}$	26%	93,6°
TOTAL	50	$\frac{50}{50}$	100%	360°

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
6.4.4 TO CONSTRUCT a circular graph from a table of data.	II	- The pupil may construct his graph in two ways: - from a table of data which is provided, or - from a table of data which he has constructed himself. - The main elements of a circular graph are: - the title or name of the graph; - the sectors of the circle whose sizes are determined by the ratio of the data for a single item to the total data; - the identification of each sector of the circle with the information needed to make the graph comprehensible by means of: - appropriate labels, - percentages, ratios, or frequencies, - etc. - It is important that pupils be required to present data carefully and to label the circular graph accurately. They must be made aware that the accurate labelling of the graph is essential to its accurate interpretation. It is equally important that they identify clearly each sector of the circle. NUMBER OF MARBLES WON 

GENERAL OBJECTIVE

7

TO INTRODUCE the pupil to the mathematical interpretation of phenomena involving chance.

THEME: Probability

TIME: 7% in Secondary II

This module will enable the pupil to explore the world of probability for the first time. The teacher should present simple and realistic situations to develop an awareness of chance, its manifestations, and the false interpretations which it is often given.

In this introduction to probability, the analysis which the pupil is asked to make of the situations presented to him should involve his intuition, his imagination, and his perception of the phenomena under study. In this analysis, the qualitative rather than the quantitative aspects of these phenomena should be stressed, particularly in the first cycle of secondary school.

The usefulness of applying the knowledge acquired in areas such as counting, multiplication, fractions, and graphic representations using tree-diagrams must also be taken into account. In this respect, the teacher can use probability to integrate several mathematical concepts.

The study of probability will gradually provide the pupil with models which will enable him to describe, to understand, and even to predict real events. It is through events occurring in his everyday environment that the pupil should first be brought to see the usefulness of the study of probability in better perceiving and understanding the real world.

TERMINAL OBJECTIVE 7.1 TO CALCULATE the probability of an outcome in a random experiment.

The mystery surrounding the concept of probability can itself provide an excellent source of motivation. The teacher should be aware that, although probability deals with a measure of uncertainty, there are numerous activities which may stimulate or create an interest in its study.

In order not to discourage the pupil, the teacher should begin with simple situations and gradually increase the level of difficulty. Above all, he should see to it that the pupil first conducts a thoughtful examination of the situation presented to him. The teacher should not overlook the fact that it is often very difficult for the pupil to understand the material which describes the situation.

In elementary school, the pupil first attempted to explore probability. The aim was to enable him:

> "To estimate and verify experimentally certain cases of known probability (intuitively)."

In secondary school, the pupil goes beyond the exploration stage to calculate the probability of an outcome.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
7.1.1 TO IDENTIFY some random experiments.	II	- A "random experiment" is an experiment whose outcome is determined by chance. It is known, however, that this outcome is one of a "universe of possible outcomes" often denoted by Ω, which is read as "omega." - Here are some examples of random experiments: - the tossing of a thumbtack; - the rolling of a die; - the tossing of a coin; - the drawing of a red marble out of a bag containing 2 blue ones, 3 green ones, and 5 red ones; - the drawing of the queen of spades from a deck of 52 cards; - the spinning of a needle on a disk with 4 congruent sectors: a green one, a white one, a blue one, and a yellow one; - etc. - It must be noted that the type of experiment to be conducted depends on the question being asked. A given situation could thus give rise to several random experiments. With the rolling of a die, for example, the following questions could be asked: - What number will turn up on the upper face? - Will this number be even? - Will the number be greater than 3?
7.1.2 TO ENUMERATE some outcomes of a random experiment.	II	- The term "universe of possible outcomes" refers to all the possible outcomes of a random experiment. - Since it can be difficult to find all the elements in the universe of possible outcomes, this objective confines the pupil to finding only some of them.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		Here are some examples: - If you spin the needle on the disk to the right, in which sector will it stop? Answer: {C, M, I, E, R} . - If you spin the needle 3 consecutive times on the above disk, noting the outcome each time, what three-letter combinations can you form? Answer: {CCC, CCR, CCI, CCM, CCE, ...} . - If you throw a die, what number will turn up on the upper face? Answer: {1, 2, 3, 4, 5, 6} or (1, 2, 3, 4, 5, 6). - If you toss a coin 3 times, what are the ways in which it could fall? Answer: (TTT, TTH, THT, THH, HTT, HTH, HHT, HHH).
7.1.3 TO LIST all possible outcomes of a random experiment.	II	- This objective is intended to enable the pupil to make an exhaustive list of the results of a random experiment. - The random experiments should be limited to simple ones so as to avoid unnecessarily complicating the tallying.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		Here is an example: - Suppose you have a jar containing 4 marbles: 1 red one (R), 1 green one (G), and two blue ones (B). If you randomly draw out 2 marbles at a time, determine the outcomes from the universe of possible outcomes. Answer: (RG, RB, RB, GB, GB, BB).
7.1.4 TO ASSIGN a probability to an outcome of a random experiment.	II	- This is a simple matter of conducting a random experiment and establishing ratios. - The ratio determines the probability of the occurrence of this favorable outcome. Let: - P be the probability; - s be the number of favorable outcomes of an experiment; - n be the number of possible outcomes of the same experiment; Example: Find the probability of drawing a five of hearts from a deck of 52 cards. Answer: Given $s = 1$, $n = 52$ then $P = \frac{s}{n} = \frac{1}{52}$.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES

Example: By rolling an ordinary die 20 times we obtain the following table of frequencies:

FREQUENCY OF OUTCOMES

Frequencies	3	5	4	3	4	1
Face	⚀	⚁	⚂	⚃	⚄	⚅

We also obtain the probability, calculated experimentally, of turning up a given face.

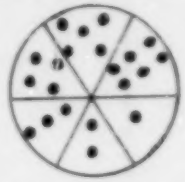
The following ratios $\frac{s}{n}$ can be established:

$$\frac{3}{20}, \frac{5}{20}, \frac{4}{20}, \frac{3}{20}, \frac{4}{20}, \text{ and } \frac{1}{20}.$$

The probability, calculated experimentally, of obtaining a given face is $P = \frac{s}{n}$:

$$\frac{3}{20}, \frac{5}{20}, \frac{4}{20}, \frac{3}{20}, \frac{4}{20}, \text{ and } \frac{1}{20}.$$

Obviously the greater the number of times the die is rolled, the closer the outcome will be to the mathematical probability. If the disk with congruent sectors is used to represent the die, measuring with a protractor makes it possible to obtain a probability which is close to the mathematical probability.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		
		- At this point it would be useful for the teacher to distinguish between "mathematical probability" and "experimental probability."
		Example: If a bag has 3 red marbles and 4 yellow ones in it, what is the probability of choosing a red marble on drawing 42 times?
		- The probability of drawing a red marble from the bag is $\frac{3}{7}$.
		- In theory, a red marble should be drawn 18 times out of every 42 tries: $42 \times \frac{3}{7} = 18.$
		- The probable frequency of this outcome could be: 16, 17, 18, 19, or 20. It is reasonable to assume that the frequency obtained experimentally will, in favorable circumstances, approach the mathematical frequency. It would therefore be possible to have the following experimental probabilities: $\frac{16}{42}, \frac{17}{42}, \frac{18}{42}, \frac{19}{42}$, or $\frac{20}{42}.$
		- The greater the number of trials, the greater tendency the experimental frequency will have to approach the mathematical frequency, and, the greater tendency the experimental probability will have to approach the mathematical probability.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		- Here is another example: Consider the roulette wheel pictured here. It is possible to try to determine the probability of the needle stopping at each of the sectors a, b, c, d, and e by turning it a certain number of times and noting at which sector it stops each time. It is also possible to measure each sector with a protractor and to establish the fact that the largest sector of the circle has the greatest chance, the smallest sector has the least chance, and congruent sectors have equal chances of being stopped at. - It should be pointed out to the pupil that during an experiment, s cannot be greater than n ($s \leq n$) and can never be negative ($s \geq 0$). - The pupil should be able to deduce that $0 \leq P \leq 1$. It is not necessary that the pupil express this fact using the symbols noted here, as long as he notes that: the probability of an outcome is always greater than or equal to zero and smaller than or equal to 1. - The importance of using simple and concrete situations cannot be overemphasized.

TERMINAL OBJECTIVE 7.2 TO CALCULATE the probability of an event.

It is important that a clear distinction be made between the concept of event and the concept of outcome. A coin-flipping experiment will be used to clarify the information below.

On looking at a coin, we see that it has two faces, one called "heads" (H) and the other "tails" (T). If the coin is flipped in the experiment, two outcomes are possible, either "heads" (H) or "tails" (T).

The set formed by all the possible outcomes is known as the "universe of possible outcomes," and in this case would be the set $\Omega = \{H, T\}$.

Since an event is a subset of the universe of possible outcomes, four subsets can be identified:

$\emptyset$, $\{H\}$, $\{T\}$, and $\{H, T\}$.

It is possible to state that:

"Any outcome is an event."

In the notes which follow, an "outcome" will sometimes be referred to as an "elementary event."

In the preceding example, it can be noted that:

$$\emptyset \subset \Omega$$

$$\{H\} \subset \Omega$$

$$\{T\} \subset \Omega$$

$$\{H, T\} \subset \Omega$$

$$P(\emptyset) = \frac{0}{2} = 0$$

$$P(\{H\}) = \tfrac{1}{2}$$

$$P(\{T\}) = \tfrac{1}{2}$$

$$P(\{H, T\}) = P(\{H\}) + P(\{T\}) + \tfrac{1}{2} + \tfrac{1}{2} = 1$$

$$P(\{H\}') = P(\{T\}) = \tfrac{1}{2}$$

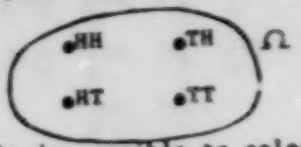
$$P(\{T\}') = P(\{H\}) = \tfrac{1}{2}$$

$$P(\emptyset') = P(\{H, T\}) = 1$$

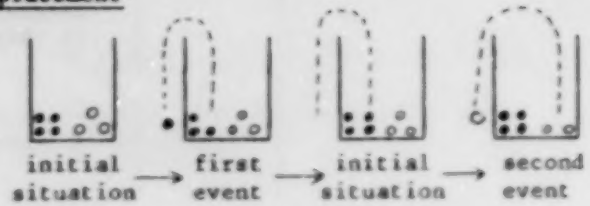
$$P(\{H, T\}') = P(\emptyset) = 0$$

The preceding brief notes show that it is possible to explore a simple situation in depth. It is not, however, advisable to go this far with the pupil. The purpose of these notes is simply to establish the relationships between the different concepts touched upon in Terminal Objectives 7.1 and 7.2.

The concepts of probability presented in this section should be taught on the basis of concrete and very simple situations so as not to add to the difficulties that the pupil has to overcome. In the second cycle of secondary school, he will have to organize his knowledge of probability more systematically. He will thus be better equipped to deal with a universe in which uncertainty plays such a large part.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
7.2.1 TO DESCRIBE an event.	II	- In a random experiment, any subset of the set of possible outcomes is called an event. - In a sense, an outcome is an event because it is an element in this set of possible outcomes. Here is an example: If you flip a coin twice, the universe of possible outcomes can be represented by:  and it is possible to select arbitrarily the event A = {HH, HT}. It can be noted that event A is a subset of the universe of possible outcomes Ω. In this example, there are 2^4 events (16). - The rolling of a die gives this set of possible outcomes: 1, 2, 3, 4, 5, 6. However simple this set may seem to be, it is possible to form 64 subsets from its elements. This rapid increase in the number of subsets or events is justification for limiting the situations to be studied to simple ones.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
7.2.2 TO FIND the set of all possible outcomes related to a simple random experiment.	II	- In the first example given in Objective 7.2.1, the universe of possible outcomes Ω is: $\{HH, HT, TH, TT\}$. It is thus possible to determine 16 subsets of Ω, each of which is an event. Here is a list of these events: $\{\}$ $\{HH\}$ $\{HH, HT\}$ $\{HH, HT, TH\}$ $\{HH, HT, TH, TT\}$ $\{HT\}$ $\{HH, TH\}$ $\{HH, HT, TT\}$ $\{TH\}$ $\{HH, TT\}$ $\{HH, TH, TT\}$ $\{TT\}$ $\{HT, TH\}$ $\{HT, TH, TT\}$ $\{HT, TT\}$ $\{TH, TT\}$ - The set of events related to this random experiment is: $\mathcal{P}(\Omega) = \{\{\}, \{HH\}, \{HT\}, \{TH\}, \{TT\}, \{HH, HT\}, \{HH, TH\}, \{HH, TT\}, \{HT, TH\}, \{HT, TT\}, \{TH, TT\}, \{HH, HT, TH\}, \{HH, HT, TT\}, \{HH, TH, TT\}, \{HT, TH, TT\}, \{HH, HT, TH, TT\}\}$
7.2.3 TO IDENTIFY some complementary events.	II	- Two events could be said to be complementary if they have no elements in common and if the elements of the two events together give the universe of possible outcomes. In other words, if A and B are two events in the universe of possible outcomes Ω, they are complementary if they simultaneously respect the following two conditions: $A \cap B = \emptyset$ $A \cup B = \Omega$.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		Here is an example: Let $\{1, 2, 3, 4, 5, 6\}$ be the set of all possible outcomes from the rolling of a die. Here are some of the events drawn from this experiment: with no elements $\{\ \}$ with one element $\{1\}, \{2\}, \{3\}, \{4\}, \ldots$ with two elements $\{1, 2\}, \{1, 3\}, \{1, 4\}, \ldots$ with three elements $\{1, 2, 3\}, \{1, 2, 4\}, \ldots$ with four elements $\{1, 2, 3, 4\}, \{1, 2, 3, 5\}, \ldots$ with five elements $\{1, 2, 3, 4, 5\}, \{1, 2, 3, 4, 6\}, \ldots$ with six elements $\{1, 2, 3, 4, 5, 6\}$. The complement of a subset is made up of all the elements not included in this subset. - All the elements not included in $\{1, 2, 5\}$ form the set $\{3, 4, 6\}$. $\{3, 4, 6\}$ and $\{1, 2, 5\}$ are complementary sets. - The complement of the set $\{3, 4\}$ is $\{1, 2, 5, 6\}$.
7.2.4 TO IDENTIFY some independent and some dependent events.	II	- Consider the following situation: in a box, there are 3 white marbles and 4 black ones. Two cases can occur. 1. <u>case with replacement</u> 

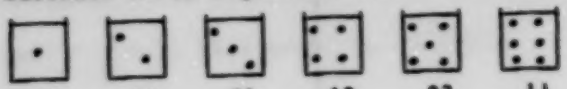
INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		In this situation, the two events are independent, because after the first event has occurred, the marble is replaced in the box before proceeding to the second event.
		2. <u>case without replacement</u>
		initial situation / first event / second event
		In this situation the two events are dependent because the second event is not based on the initial situation, but on the situation as modified by the first event.
7.2.5 TO ASSIGN a probability to an event.	II	- Knowing the probability of the elementary outcomes, it is possible to calculate the probability of more complex events. If the probability of the elementary events is not known, an experiment can be performed with a view to assigning an experimental probability. It is advisable to make a definite number of trials in order to facilitate computation.
		Here is an example:
		In an experiment involving the rolling of a die, the set of possible outcomes is $\{1, 2, 3, 4, 5, 6\}$, and if P (A) indicates the probability of event A's occurence, it is possible to write:
		$$P(\{1\}) = P(\{2\}) = P(\{3\}) = P(\{4\}) = P(\{5\}) = P(\{6\}).$$
		which is to say that all the elementary events have an equal probability of occurring.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES

It is then possible to state that:

$$P(\{3, 4\}) = P(\{3\}) + P(\{4\})$$
$$P(\{1, 3, 5, 6\}) = P(\{1\}) + P(\{3\}) + P(\{5\}) + P(\{6\})$$
etc.

Here are the outcomes of an experiment after 100 rolls.



19 10 24 13 23 11

The experimental probability is thus

$$\frac{19}{100}, \frac{10}{100}, \frac{24}{100}, \frac{13}{100}, \frac{23}{100}, \text{ and } \frac{11}{100}.$$

Hence $P(\{1, 3, 5, 6\}) = \frac{19}{100} + \frac{24}{100} + \frac{23}{100} + \frac{11}{100} = \frac{77}{100}.$

and $P(\{1, 5\}) = \frac{19}{100} + \frac{23}{100} = \frac{42}{100}.$

- In general, for an event denoted A, its complement will be denoted A' and it is possible to state the following rule:
$$P(A') = 1 - P(A).$$

- An event can be either certain, probable, or impossible.

- An event is said to be:
  - "certain" if the probability of its occurring is 1;
  - "probable" if the probability of its occurring is between 0 and 1;
  - "impossible" if the probability of its occurring is nil.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		Example: Find the probability of turning up a 7 on a die which has 6 faces numbered from 1 to 6. P(7) = 0; this event is "impossible." Example: Find the probability of drawing a red marble from a bag containing 5 red marbles. P(red) = $\frac{5}{5}$ = 1; this event is certain. Example: Find the probability of turning up an odd number on a die which has 6 faces numbered from 1 to 6. P(odd) = $\frac{3}{6} = \frac{1}{2}$. - CALCULATE the probability of two independent events occurring at the same time in a simple experiment. The probability here is equal to the product of the probabilities of each of the two events. Example: The probability of turning up 2 three's in rolling a pair of dice is: $P(2) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. - CALCULATE the probability of two dependent events occurring in succession in a simple experiment. The probability here is equal to the product of the probabilities obtained in each successive trial.

INTERMEDIATE OBJECTIVES	LEVELS	NOTES
		Example: If a bag contains 2 blue marbles, 2 red ones, 2 green ones, 2 yellow ones, and 2 black ones, the probability of drawing a black marble and then a red one is: $P(\text{black, red}) = \frac{2}{10} \times \frac{2}{9} = \frac{1}{5} \times \frac{2}{9} = \frac{2}{45}$.
		- The sum of all the probabilities of the possible elementary events (outcomes) in a simple random experiment is 1.
		Example: If a bag contains 3 red marbles, 4 green ones, and 7 black ones, determine the probability of choosing one of these three colours. The probability of drawing: - a red marble is: $P(R) = \frac{3}{14}$ - a green marble is: $P(G) = \frac{4}{14}$ - a black marble is: $P(B) = \frac{7}{14}$ and the probability of choosing 1 of these three colours is: $P(A, B, C) = P(A) + P(B) + P(C) = \frac{3}{14} + \frac{4}{14} + \frac{7}{14} = \frac{14}{14} = 1$.

3

EDUCATIONAL EVALUATION

This section of the Curriculum Guide states the criteria and techniques which are applicable to the evaluation of learning and which are likely to facilitate the assessment of the extent to which course objectives have been achieved.

It is important to recall the responsibility of the teacher in this regard: "evaluation takes place first of all in the classroom and ... the prime responsibility is the teacher's."[1] One of the teacher's main concerns should be to integrate educational evaluation into a dynamic learning experience for the pupil.

> "The evaluation of a pupil's learning should be concerned with those instructional objectives which are the subject of his daily efforts. If the evaluation is to bear upon the instructional objectives, it should also be appropriate to the mental processes required by the instruction. This is a question of both quality of education and of justice for the pupil. It would demonstrate incoherence if, for example, the instruction involved comprehension and analysis, while the evaluation, in its turn, was concerned only with simple memorization."[2]

Educational evaluation must always have a reason:

> "Whatever may be the reasons for evaluating a pupil's learning, the teacher must always be aware of the underlying reasons for every evaluation operation he undertakes. Is it to establish the pupil's individual progress toward objectives which have been set? Is it to make a general assessment of the progress of the whole school or of a group taking the course? Is it to form working sub-groups so that instruction can be better adapted to the current learning rates of each group? Is it to provide parents with information about the progress of their children? Is it to assess the effectiveness of the teaching strategies that are being used?"[3]

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1. Québec, Ministère de l'Education. The Schools of Québec: Policy Statement and Plan of Action, 1979, p. 94.
2. Québec, Ministère de l'Education. General Policy for Educational Evaluation, 1981, no. 14-7500 A, p. 10.
3. Ibid, p. 10.

Since the teacher is primarily responsible for the educational evaluation of the pupil's learning progress, it is his responsiblity to decide at what time a given measurement instrument should be used.

> "Where formative evaluation is involved, the teacher is the best situated to decide when information should be obtained. The teacher knows when the instructional activity has been completed and at what point it would be most appropriate to check if the desired pupil learning has actually taken place.
>
> "The teacher or the school principal may also decide at which times summative evaluation would be appropriate. If, for example, it is desirable to determine the level of mastery of a group of pupils for a set of concepts or skills, a group evaluation profile could be made at what is considered to be the appropriate point in the evaluation plan."[1]

In a dynamic learning process, evaluation plays a crucial role. It

- "provides information on the quality of the pupils' academic performance and general development;
- "helps in the appraisal of the quality of the pupils' learning progress;
- "helps in the diagnosis of the learning or general development problems of individual pupils or of groups of pupils;
- "leads to decisions which make it possible to modify learning activities." [2]

Assignments, questionnaires, tests and examinations are some of the measurement instruments frequently used in the classroom. Some teachers go further and develop observation checklists as a type of evaluation tool. Along the same lines, this Curriculum Guide provides forms which are designed to facilitate the carrying out of formative evaluation supported by a criterion-referenced interpretation (Form A), while paying due attention as well to summative evaluation, in which norm-referenced interpretation is used (Form B).

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1. Ibid, page 11.
2. S.G.M.E. La mesure critériée et la mesure normative, 1976. This citation is a free translation.

Whether or not the forms provided in this document are used, as a general rule the teacher should not take the scores which have been used in a formative evaluation as the basis for a norm-referenced interpretation. These scores are indices which are intended to enable him to take remedial action and should not normally appear on the pupil's official school report.

The purpose of the summative evaluation recorded on the pupil's report is to assess the extent to which he has mastered a set of learning objectives. It complements formative evaluation, adding data concerning time (retention) and the accumulation of knowledge (which can in some cases increase the level of mastery).

This type of evaluation involves a minimum standard (the passing mark). Because of these features, it lends itself to criterion-referenced interpretation and can sometimes be used for this purpose.

The following four-part table situates the dynamic process of evaluating a pupil's learning and of interpreting that evaluation within the context of the instructional approach detailed in the preceding paragraphs.

		INTERPRETATION	
		NORM-REFERENCED	CRITERION-REFERENCED
EVALUATION	Formative	FOR OCCASIONAL USE	FOR FREQUENT USE
	Summative	FOR FREQUENT USE	FOR OCCASIONAL USE

Form A makes it possible to effect a formative evaluation of learning. Results can be measured through diagnostic tests, which ensure measurement of the extent to which each terminal objective has been mastered. These tests lend themselves to a criterion-referenced interpretation of the learning progress of each pupil and help the teacher take the necessary remedial measures. Such tests are for diagnostic purposes only and should not be used for decisions regarding promotion. The degree of mastery of each general objective is evaluated separately and is expressed as a score out of 100, which is then weighted by the factor n. Weighting is necessary so that the mark reflects the relative importance accorded to each general objective. The value of n can be determined using as a basis the percentages shown in the table(s) in Section 5.2 of the program. These values range from 0 to 100%. The mark obtained in this way is entered in the appropriate box in Form B.

Since a pupil's progress is not always directly proportionate to his scores, the teacher is strongly encouraged to carry out a co-evaluation in which both he and the pupil assess the latter's general progress with respect to generally accepted criteria: effort, perseverance, quality and quantity of work, participation, and so on. The result is expressed using a scale such as the one which follows:

5 excellent

4 very good

3 good

2 average

1 below average

0 unsatisfactory.

The average of these co-evaluations is entered in the corresponding box on Form B. After Form A has been completed, the teacher can use it as a tool to carry out a criterion-referenced evaluation of each pupil's learning.

Form B makes it possible to compile and compare the marks which have been obtained through co-evaluations and through examinations, and which have been calculated on each of the A-type forms. The average of the marks in the co-evaluation column and the sum of the weighted marks in the examination column makes it possible to form an overall picture of the pupil's progress.

The mark obtained on the final examination, the content of which must cover all the general objectives of the program, makes it possible to ascertain whether or not the pupil has been able to synthesize the concepts studied. The final examination must be prepared and administered by the educational body (the school board, the regional school board, or the ministère de l'Education) which is responsible for implementing this program. A mark out of 50 for tests, combined with a mark out of 50 for the final examination, provides a score expressed out of 100 which makes it possible to make a general assessment of the pupil's learning progress. The two marks appearing at the bottom of the form are important elements in summative evaluation with regard to the academic knowledge acquired by the pupil and in decisions concerning his promotion.

The following pages illustrate Forms A and B, which can be used for the entire program for the first cycle of secondary school.

First Name:	Surname:
Program for: First Cycle, Sec. I	

GENERAL OBJECTIVE	No.	TO ENCOURAGE the pupil to apply his knowledge of natural numbers or integers.

	TERMINAL OBJECTIVES
1.1	TO SOLVE problems dealing with numeration.
1.2	TO PERFORM basic operations on natural numbers using various algorithms.
1.3	TO PERFORM basic operations on integers using various algorithms.

Co-evaluation
Diagnostic Test
Examination

A

n = 0,18

$\frac{x\ n}{100}$

Average

Co-evaluation: 5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

First Name:	Surname:
Program for: First Cycle, Sec. I	

GENERAL OBJECTIVE	No. 2	TO ENCOURAGE the pupil to apply his knowledge of rational numbers.

	TERMINAL OBJECTIVES	Co-evaluation	Diagnostic Test	Examination
2.1	TO PERFORM basic operations on fractions using various algorithms.			
2.2	TO SOLVE problems representing real-life situations involving decimal			
	numbers.			
2.3	TO SOLVE problems dealing with buying/selling using the concept of			
	percentage.			
2.4	TO SOLVE problems involving proportions.			
2.5	TO FIND the square root of a number in certain situations.			

Co-evaluation:
5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

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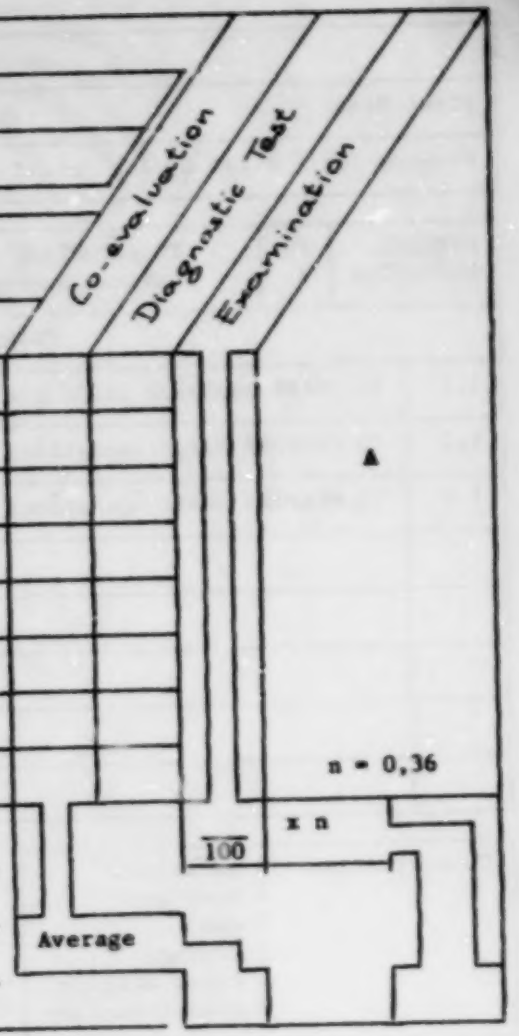


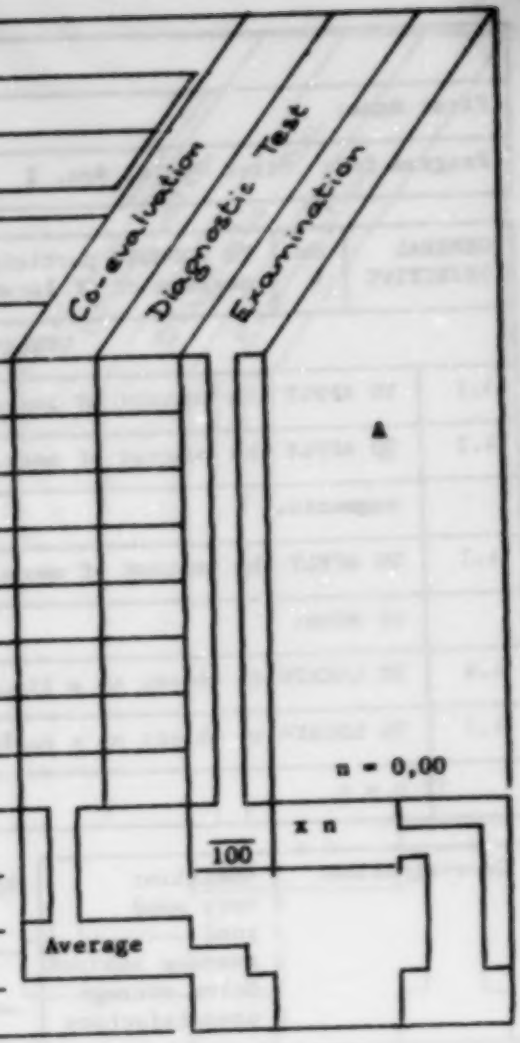
First Name:	Surname:
Program for: First Cycle, Sec. I	

GENERAL OBJECTIVE	No. 3	TO PROVIDE the pupil with basic training in algebra.

	TERMINAL OBJECTIVES
3.1	TO PERFORM basic operations on simple algebraic expressions.
3.2	TO SOLVE problems involving common formulas, or simple equations or
	inequations in one variable.

Co-evaluation: 5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____



First Name:	Surname:
Program for: First Cycle, Sec. I	

GENERAL OBJECTIVE	No. 4	TO PROMOTE participation in activities involving the concept of measure or of location.

	TERMINAL OBJECTIVES	Co-evaluation	Diagnostic Test	Examination
4.1	TO APPLY the concept of angle measure.			
4.2	TO APPLY the concept of measure in situations involving the measurement of			
	segments.			
4.3	TO APPLY the concept of measure in situations involving the measurement			
	of area.			
4.4	TO LOCATE an object on a line.			
4.5	TO LOCATE an object on a surface.			

A

n = 0,11

$\frac{}{100}$ x n

Average

Co-evaluation:	5 excellent 4 very good 3 good 2 average 1 below average 0 unsatisfactory

Remarks: _____

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_____

First Name:	Surname:
Program for: First Cycle, Sec. I	

GENERAL OBJECTIVE	No.	TO DEVELOP in the pupil the habit of using geometric concepts, relations, and properties.

	TERMINAL OBJECTIVES
5.1	TO REPRODUCE a plane figure using geometric transformations.
5.2	TO IDENTIFY the image of a relation, given this relation.
5.3	TO CONSTRUCT various plane geometric figures.
5.4	TO IDENTIFY the various properties of triangles and quadrilaterals.
5.5	TO SOLVE problems in everyday life using units of length or of area.

Co-evaluation
Diagnostic Test
Examination

A

n = 0,27

$\frac{\text{x n}}{100}$

Average

Co-evaluation: 5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

_____

_____

_____

First Name:	Surname:
Program for: First Cycle, Sec. I	

GENERAL OBJECTIVE	No. 6	TO INTRODUCE the pupil to the language and to the application of elementary statistics.

	TERMINAL OBJECTIVES
6.1	TO COLLECT information from different kinds of graphs illustrating familiar
	situations.
6.2	TO CONSTRUCT a table of statistical data drawn from familiar situations.
6.3	TO CONSTRUCT broken-line graphs or bar graphs to illustrate certain
	familiar situations.
6.4	TO CONSTRUCT a circular graph to illustrate certain familiar situations.

Co-evaluation
Diagnostic Test
Examination

A

n = 0,08

$\frac{x\ n}{100}$

Average

Co-evaluation: 5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

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_____

First Name:	Surname:
Program for: First Cycle, Sec. I	

GENERAL OBJECTIVE	No. 7	TO INTRODUCE the pupil to the mathematical interpretation of phenomena involving chance.

	TERMINAL OBJECTIVES
7.1	TO CALCULATE the probability of an outcome in a random experiment.
7.2	TO CALCULATE the probability of an event.

Co-evaluation
Diagnostic Test
Examination

A

n = 0,00

$\frac{x\ n}{100}$

Average

Co-evaluation: 5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

First Name: Surname:

Program for: First Cycle, Sec. I

	GENERAL OBJECTIVES	Co-evaluation	Diagnostic Test	Examination
1	TO ENCOURAGE the pupil to apply his knowledge of natural numbers or integers.			
2	TO ENCOURAGE the pupil to apply his knowledge of rational numbers.			
3	TO PROVIDE the pupil with basic training in algebra.			
4	TO PROMOTE participation in activities involving the concept of measure or of location.			
5	TO DEVELOP in the pupil the habit of using geometric concepts, relations, and properties.			
6	TO INTRODUCE the pupil to the language and to the application of elementary statistics.			
7	TO INTRODUCE the pupil to the mathematical interpretaion of phenomena involving chance.			

B

100

100

200

100

Average

Co-evaluation: 5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

_____

_____

_____

First Name:	Surname:
Program for: First Cycle, Sec. II	

GENERAL OBJECTIVE	No. 1	TO ENCOURAGE the pupil to apply his knowledge of natural numbers or integers.

	TERMINAL OBJECTIVES
1.1	TO SOLVE problems dealing with numeration.
1.2	TO PERFORM basic operations on natural numbers using various algorithms.
1.3	TO PERFORM basic operations on integers using various algorithms.

Co-evaluation
Diagnostic Test
Examination
A
n = 0,11
x n
100
Average

Co-evaluation: 5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

First Name:	Surname:
Program for: First Cycle, Sec. II	

GENERAL OBJECTIVE	No. 2	TO ENCOURAGE the pupil to apply his knowledge of rational numbers.

	TERMINAL OBJECTIVES	Co-evaluation	Diagnostic Test	Examination
2.1	TO PERFORM basic operations on fractions using various algorithms.			
2.2	TO SOLVE problems representing real-life situations involving decimal numbers.			
2.3	TO SOLVE problems dealing with buying/selling using the concept of percentage.			
2.4	TO SOLVE problems involving proportions.			
2.5	TO FIND the square root of a number in certain situations.			

A

n = 0,16

$\frac{}{100}$ x n

Average

Co-evaluation: 5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

First Name:	Surname:
Program for: First Cycle, Sec. II	

			Co-evaluation	Diagnostic Test	Examination
GENERAL OBJECTIVE	No. 3	TO PROVIDE the pupil with basic training in algebra.			
		TERMINAL OBJECTIVES			
3.1		TO PERFORM basic operations on simple algebraic expressions.			
3.2		TO SOLVE problems involving common formulas, or simple equations or			
		inequations in one variable.			

A

n = 0,20

x n

$\frac{}{100}$

Average

Co-evaluation:
5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

_____

_____

_____

First Name:	Surname:
Program for: First Cycle, Sec. II	

GENERAL OBJECTIVE	No. 4	TO PROMOTE participation in activities involving the concept of measure or of location.

	TERMINAL OBJECTIVES
4.1	TO APPLY the concept of angle measure.
4.2	TO APPLY the concept of measure in situations involving the measurement of
	segments.
4.3	TO APPLY the concept of measure in situations involving the measurement
	of area.
4.4	TO LOCATE an object on a line.
4.5	TO LOCATE an object on a surface.

Co-evaluation / Diagnostic Test / Examination

A

$n = 0,11$

$\frac{}{100} \times n$

Average

Co-evaluation: 5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

_____

_____

_____

First Name:	Surname:
Program for: First Cycle, Sec. II	

GENERAL OBJECTIVE	No.	TO DEVELOP in the pupil the habit of using geometric concepts, relations, and properties.

	TERMINAL OBJECTIVES
5.1	TO REPRODUCE a plane figure using geometric transformations.
5.2	TO IDENTIFY the image of a relation, given this relation.
5.3	TO CONSTRUCT various plane geometric figures.
5.4	TO IDENTIFY the various properties of triangles and quadrilaterals.
5.5	TO SOLVE problems in everyday life using units of length or of area.

Co-evaluation
Diagnostic Test
Examination

A

n = 0,22

$\frac{x\ n}{100}$

Average

Co-evaluation:
5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

First Name:	Surname:
Program for: First Cycle, Sec. II	

GENERAL OBJECTIVE	No. 6	TO INTRODUCE the pupil to the language and to the application of elementary statistics.
		TERMINAL OBJECTIVES
6.1		TO COLLECT information from various diagrams illustrating familiar situations.
6.2		TO CONSTRUCT a table of statistical data drawn from familiar situations.
6.3		TO CONSTRUCT broken-line graphs or bar graphs to illustrate certain familiar situations.
6.4		TO CONSTRUCT circular graphs to illustrate certain familiar situations.

Co-evaluation
Diagnostic Test
Examination

A

$n = 0,11$

$\frac{x\ n}{100}$

Average

Co-evaluation:
5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

First Name:	Surname:
Program for: First Cycle, Sec. II	

GENERAL OBJECTIVE	No. 7	TO INTRODUCE the pupil to the mathematical interpretation of phenomena involving chance.

	TERMINAL OBJECTIVES	Co-evaluation	Diagnostic Test	Examination
7.1	TO CALCULATE the probability of an outcome in a random experiment.			
7.2	TO CALCULATE the probability of an event.			

A

n = 0,09

x n

100

Average

Co-evaluation: 5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

First Name: | Surname:

Program for: First Cycle, Sec. II

	GENERAL OBJECTIVES	Co-evaluation	Diagnostic Test	Examination
1	TO ENCOURAGE the pupil to apply his knowledge of natural numbers or integens.			
2	TO ENCOURAGE the pupil to apply his knowledge of rational numbers.			
3	TO PROVIDE the pupil with basic training in algebra.			
4	TO PROMOTE participation in activities involving the concept of measure or of			
	location.			
5	TO DEVELOP in the pupil the habit of using geometric concepts, relations, and			
	properties.			
6	TO INTRODUCE the pupil to the language and application of elementary			
	statistics.			
7	TO INTRODUCE the pupil to the mathematical interpretation of phenomens			
	involving chance.			

B

		100
		100
Average		200
		100

Co-evaluation:
5 excellent
4 very good
3 good
2 average
1 below average
0 unsatisfactory

Remarks: _____

INSTRUCTIONAL MATERIALS

INSTRUCTIONAL MATERIALS

The teacher can make the subject of mathematics meaningful by helping the pupil discover for himself and apply the following basic concepts: classification, numbers, numeration, measure, and geometric operations or forms. In such an approach, the actual use of these concepts is indeed of primary importance. It is through everyday situations, motivating learning activities, and a variety of experiments in the form of games, codes, and representations, that the pupil comes to formulate concepts in his own terms and to develop his own logical reasoning skills.

In order to enhance this process, both teacher and pupils should have access to a wide variety of readily available instructional materials. Due to the many restrictions imposed by the cost of materials, the number of users, and storage problems, this material should be organized in a pedagogically sound and physically logical manner.

This section provides a list of instructional materials. It is not exhaustive and does not include either written materials such as textbooks, exercise books, answer books, teachers' guides, or audio-visual materials.

CLASSROOM MATERIALS

QUANTITY	DESCRIPTION
	Measurement of Angle and Length
1	Wooden blackboard compass, 420 mm long, double pivot, rubber/steel.
1	Flexible curve, lead bar encased in vinyl mounting, 600 mm long.
1	Plastic or wooden blackboard set square, 30°-60°, two-sided, 500 mm high, graduated, with handle.
1	Plastic or wooden blackboard set square, 45°, two-sided, 400 mm high, graduated, with handle.
1	Plastic or wooden blackboard protractor, 180°, two-sided, 500 mm baseline.
1	Plastic or wooden blackboard ruler, 64 mm wide, 1000 mm long, graduated, with handle.
1	Plastic or wooden T-square, 900 mm long, with handle.
3	Clinometer, revolving type, plastic and lead.
3	Plastic or wooden calibrating compass, 300 mm long for inside and outside measurements.
3	(0-25)mm or 0,01 mm stainless steel micrometer.
3	Sliding calipers, 900 to 1000 mm long, graduated, for inside and outside measurements.
3	Plastic trundle wheels, 1000 mm circumference, graduated, clicking mechanism, handle.
3	Fibreglass tape measures, 20 000 mm long, graduated in mm.

QUANTITY	DESCRIPTION
	Volume and Surface
1	Plastic centimetre cube (1000), 1 g mass, faces of assorted colours, interlocking.
1	Clear plastic litre container, cube with 100 mm edge, graduated in millimetres.
35	Transparent plastic grids, 200 mm x 200 mm, surface measurement.
1	Dismountable meter cube frame, 3 rods painted in decimetres.
1	Plastic beaker, 1000 ml, graduated in 50 ml, with handle and pouring lip.
1	Plastic hollow blocks, 150 mm high, with filling holes, set of 14.
	Geometry and Algebra
1	Centre angle and inscribed angle, magnetic material.
17	Geoboards, 300 mm x 300 mm, pegs, elastic bands of assorted colours.
1	Parallel rule cut by a transversal, magnetic material.
1	Pythagorean theorem, magnetic material.

QUANTITY	DESCRIPTION
	Supplementary Materials
1	Compass, Brunton "Cadet"type, graduations 0-360.
35	Calculators.
1	Numerical straightedge, magnetic blackboard, 200 mm wide, 1500 mm long, disks.
1	Initiation to probability and statistics kit.
1	Inclinable level, horizontal angles, magnification 18x.
1	Compensating polar planimeter, plastic, magnifying lens.
10	Polyforms, assembly of rigid cardboard boxes, elastic bands.
10	Structure building, plastic straws, junction knots.
10	Auto Correct Art, case and sets.
10	Veritech, self-correcting, box.
35	Mirrors (Mira or other).
1	Multi-base blocks (1 set).
2	Logic grids and pattern blocks.
2	Trimath and quadrimath.
1	Polar coordinate and rectangular coordinate board (two-sided).
	A variety of mathematical games.
8	Micro-computers.

MATERIAL SUPPLIED BY THE SCHOOL

QUANTITY	DESCRIPTION
	Wood glue, white, 5 kg container.
	Set of cardboard boxes, assorted colours.
	Graph paper squared in millimetres.
35 per classroom-workshop	All-purpose scissors, steel, 200 mm long, right-handed, left-handed.

MATERIAL BOUGHT BY THE PUPIL

QUANTITY	DESCRIPTION
	Pencil compass, brass, 150 mm long, blocking screw or clamp.
	Professional set square, transparent plastic, 150 mm high, 30°-60°, two-sided.
	Professional set square, transparent plastic, 150 mm, 45°, two-sided.
	Plastic angle protractor, 150 mm, semi-circular, double scale, sighting mark.
	Plastic or wooden ruler, 300 mm, graduated in mm.
	Calculator.

BIBLIOGRAPHY

The bibliography included here is not exhaustive, but it does provide additional sources for other possible activities.

As a general rule, books recognized by the ministère de l'Éducation and school board materials (activity cards) are not listed.

All the bibliographical entries are grouped into subsections corresponding to each of the program's general objectives.

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